ISE 315: Engineering Statistics

Lecture 3: Practice on Estimators

Instructor: Mansur M. Arief, PhD
Industrial and Systems Engineering, KFUPM

Office: 22-219 — Email: mansur.arief@kfupm.edu.sa

Based on Montgomery & Runger, Applied Statistics and Probability for Engineers, 6th Ed.

Lecture 3

Practice on Estimators

Lecture 3 Outline

- Discussion on estimation concepts
- Practice problems on estimators
- Q & A on homework or mini quizzes

Share **two estimation concepts** you remember from last lecture!

Scan the QR or visit **pe.app/ise315**!

A random sample of 36 observations has been drawn from a normal distribution with mean 50 and standard deviation 12. Find the probability that the sample mean is in the interval $47 \leq X \leq 53$. Is the assumption of normality important? Why?

Problem 1 (7.S11)

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How do we solve this again?

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- The probability $\bar{X} \in [47, 53]$ is

$$\begin{aligned} P(47 \leq \bar{X} \leq 53) &= P\left(\frac{47-50}{2} \leq Z \leq \frac{53-50}{2}\right) \\ &= P(-1.5 \leq Z \leq 1.5) \\ &= 0.9332 - 0.0668 = 0.8664 \end{aligned}$$

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- No, because CLT gives us that $\bar{X} \sim N(50, 2)$ approximately.

Consider two estimators for the population mean μ: $\hat{\theta}_1 = \bar{X}$ and $\hat{\theta}_2 = \frac{n}{n+1}\bar{X}$. For a sample of size $n = 9$ from $N(\mu, \sigma^2)$ with $\sigma^2 = 9$, compute the bias, variance, and MSE of each estimator. If $\mu = 1$, which is better? If $\mu = 10$, which is better?

Problem 2

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- Recall: $\text{MSE}(\hat{\theta}) = \text{Var}(\hat{\theta}) + [\text{Bias}(\hat{\theta})]^2$

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- Recall: $\text{MSE}(\hat{\theta}) = \text{Var}(\hat{\theta}) + [\text{Bias}(\hat{\theta})]^2$
- $\text{Bias}(\hat{\theta}) = \mathbb{E}[\hat{\theta}] - \theta$

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$$
\begin{aligned}
\text{MSE}(\hat{\theta}_2) < \text{MSE}(\hat{\theta}_1) &\implies \frac{\mu^2}{100} + 0.81 < 1 &&\implies \frac{\mu^2}{100} < 0.19 \\
&\implies \mu^2 < 19 &&\implies \|\mu\| < 4.36
\end{aligned}
$$

A semiconductor manufacturer samples 100 chips, classifying each as defective ($X_i = 1$) or nondefective ($X_i = 0$). The sample fraction defective is $\hat{P} = \frac{X_1 + X_2 + \cdots + X_{100}}{100}$. What is the sampling distribution of $\hat{P}$?

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Problem 3 (7.S12)

What is our population distribution (for X_i)?

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Step 3: By CLT (since $n = 100$ is large): $\boxed{\hat{P} \overset{\text{approx}}{\sim} N\left(p, \frac{p(1-p)}{n}\right)}$

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Some Useful Properties of Expectation and Variance

From ISE 205

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Consider two independent random variables

- $X \sim p_1$ with $\mathbb{E}[X] = \mu_1, \mathrm{Var}(X) = \sigma_1^2$
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- $Y \sim p_2$ with $\mathbb{E}[Y] = \mu_2, \mathrm{Var}(Y) = \sigma_2^2$
- Linearity of expectation:
  - Addition with random variables: $\mathbb{E}[X \pm Y] = \mathbb{E}[X] \pm \mathbb{E}[Y] = \mu_1 \pm \mu_2$
  - Addition with constants: $\mathbb{E}[X \pm a] = \mathbb{E}[X] \pm a = \mu_1 \pm a$
  - Multiplication with constants: $\mathbb{E}[aX] = a\mathbb{E}[X] = a\mu_1$
- Properties of variance:
  - Addition with random variables: $\mathrm{Var}(X \pm Y) = \mathrm{Var}(X) + \mathrm{Var}(Y) = \sigma_1^2 + \sigma_2^2$
  - Addition with constants: $\mathrm{Var}(X \pm a) = \mathrm{Var}(X) = \sigma_1^2$
  - Multiplication with constants: $\mathrm{Var}(aX) = a^2\mathrm{Var}(X) = a^2\sigma_1^2$

Practice quiz #1 (also available on Blackboard)

Scan the QR or visit **pe.app/ise315**!

A quality engineer wants to estimate the mean tensile strength of a new alloy. From past experience, $\sigma = 8$ MPa. How large a sample is needed so that the sample mean is within 2 MPa of the true mean with 95% confidence?

Problem 4: Sample Size Determination (if time permits)

What does “within 2 MPa with 95% confidence” mean mathematically?

Two drilling methods are compared. Method A: $n_1 = 40$ wells, $\bar{X}_1 = 28.5$ days, $\sigma_1 = 6$ days. Method B: $n_2 = 36$ wells, $\bar{X}_2 = 32.8$ days, $\sigma_2 = 8$ days. Find the probability that $\bar{X}_1 - \bar{X}_2$ is within 2 days of the true difference $\mu_1 - \mu_2$.

Problem 5: Difference of Sample Means (if time permits)

What is the sampling distribution of $\bar{X}_1 - \bar{X}_2$?

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- Quiz #1 next class (5 questions about estimators 7.1-7.3, 10-15 minutes)
- Next class: Quiz #1 and Chapter 8.1