ISE 315: Engineering Statistics

Lecture 5: Statistical Intervals for a Single Sample (Part 1)

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Based on Montgomery & Runger, Applied Statistics and Probability for Engineers, 6th Ed.

Lecture 5

Statistical Intervals for a Single Sample (Part 1)

Lecture 5 Outline: (Chapter 8.1)

- Announcements:
  - HW 1 due Wednesday before class
  - HW 2 due next Wednesday before class
  - Major Exam 1 in Week 8 (Chapters 7, 8, 9, 10)
- From Point Estimates to Interval Estimates
- Confidence Interval on the Mean (σ^2 Known) (8-1.1)
- Choice of Sample Size (8-1.2)
- One-Sided Confidence Bounds (8-1.3)

Quick survey!

Scan the QR or visit **pe.app/ise315**!

Chapter 8 Concept Review

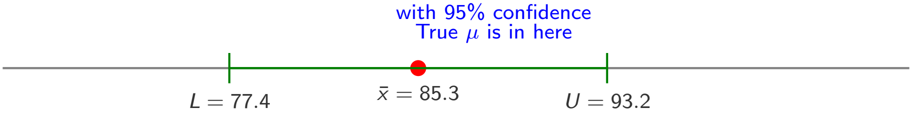
Point Estimate
$\hat{\mu} = \bar{x} = 85.3$

Add uncertainty →

Interval Estimate
$(77.4, 93.2)$

How confident?

95% confident!



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4. **Confidence Level:**
   - $1 - \alpha =$ probability the method produces an interval containing θ
   - 95% CI $\Rightarrow \alpha = 0.05$

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The margin of error combines critical value and standard error: $E = z_{\alpha/2} \times SE$

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One-sided intervals use z_α instead of $z_{\alpha/2}$ (all α on one side)

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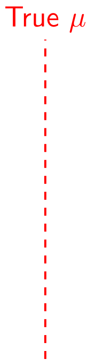
  - "The interval (L, U) was constructed using a method that captures μ 95% of the time"

Visualizing Confidence Intervals: Repeated Sampling



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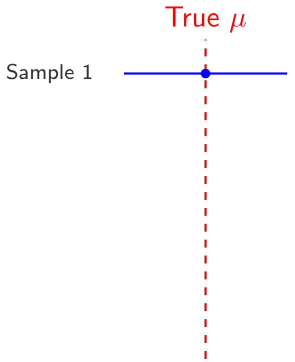

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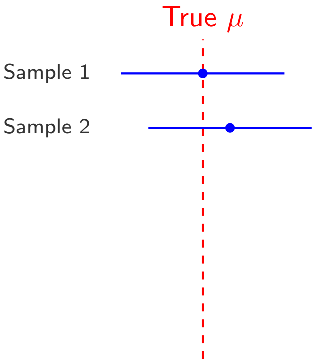

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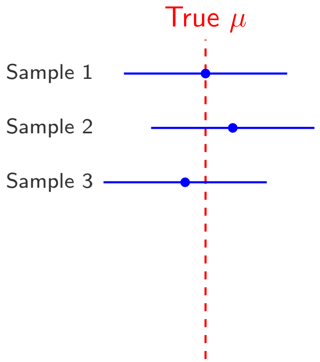

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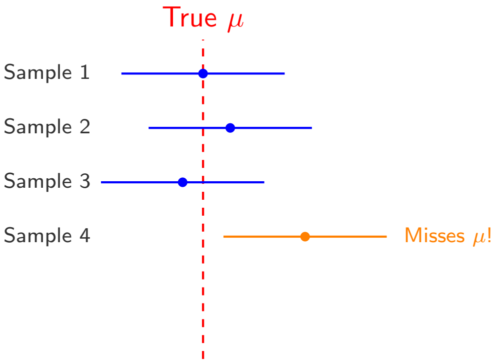

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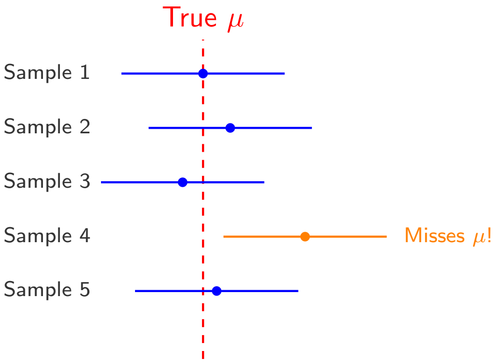

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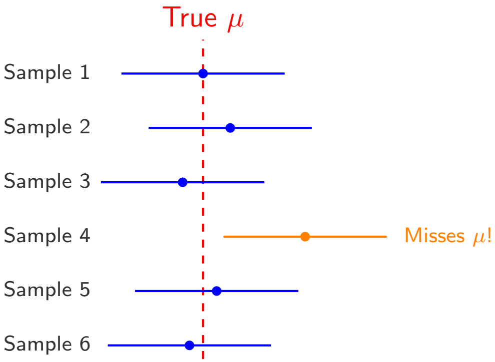

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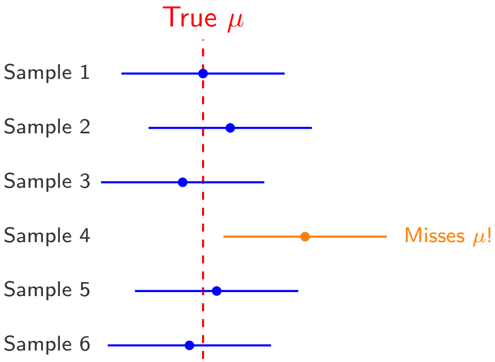

With 95% CI: about 95 out of 100 intervals capture μ

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Confidence Interval Formulas (σ Known)

Two-Sided $100(1-\alpha)\%$ **Confidence Interval for** μ**:**

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Sample Size for Desired Margin of Error E**:** $\boxed{n = \left(\frac{z_{\alpha/2}\sigma}{E}\right)^2}$ (round up!)

Common Critical Values $z_{\alpha/2}$

Confidence Level	α	$\alpha/2$	$z_{\alpha/2}$
90%	0.10	0.05	1.645
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- **Memory tip:** For 95% CI, use $z \approx 2$

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- Compare confidence intervals with different confidence levels or sample sizes

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- Calculate margin of error: $E = z \times \text{SE}$
- Construct interval: $\bar{x} \pm E$ or $\bar{x} - E$ (lower bound) or $\bar{x} + E$ (upper bound)

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- Sample size determination with different constraints

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- Sample size determination with different constraints
- Comparing precision of different confidence levels

Questions you might be asked about confidence intervals

What variants we might see?

- The population standard deviation σ might be unknown
  - Use t-distribution instead of z (HW 2 and next lecture!)
- The parameter might not be the mean μ
  - Variance $\sigma^2 \Rightarrow$ use χ^2 distribution
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Same approach:
Parameter $\rightarrow$ Sampling distribution $\rightarrow$ Get critical value $\rightarrow$ CI

A petroleum engineer measures wall thickness of $n = 25$ pipe sections. Sample mean $\bar{x} = 0.2731$ inches. Historical data shows $\sigma = 0.02$ inches. Construct a 95% CI for true mean wall thickness.

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Given:

- $n = 25$, $\bar{x} = 0.2731$, $\sigma = 0.02$
- Confidence level: 95% $\Rightarrow \alpha = 0.05 \Rightarrow z_{0.025} = 1.96$

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$$\text{SE} = \frac{\sigma}{\sqrt{n}} = \frac{0.02}{\sqrt{25}} = \frac{0.02}{5} = 0.004$$

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We are 95% confident that μ is between 0.2653 and 0.2809 inches.

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$$= \left(\frac{117.6}{15}\right)^2 = (7.84)^2 = 61.47 \approx 62 \text{ (round up!)}$$

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$$= 1850 - 1.645 \times 25 = 1850 - 41.125 = \boxed{1808.9 \text{ kg}}$$

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- **Remember**: This assumes σ is known!
  - Next lecture: What if σ is unknown? $\Rightarrow$ t-distribution

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See you next class!