ISE 315: Engineering Statistics

Lecture 7: Practice on Confidence Intervals

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Based on Montgomery & Runger, Applied Statistics and Probability for Engineers, 6th Ed.

Lecture 7

Practice on Confidence Intervals

Lecture 7 Outline

- Quick review of confidence interval formulas
- Practice problems on confidence intervals
- Q & A on homework or quizzes

Quick Review: Confidence Interval Formulas

Chapter 8

Parameter	Two-Sided CI Formula	When to Use
Mean μ (σ known)	$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$	σ is given/historical
Mean μ (σ unknown)	$\bar{x} \pm t_{\alpha/2,n-1} \cdot \frac{s}{\sqrt{n}}$	Only sample std dev s
Variance σ^2	$\left[\frac{(n-1)s^2}{\chi^2_{\alpha/2,n-1}}, \frac{(n-1)s^2}{\chi^2_{1-\alpha/2,n-1}}\right]$	Need CI for spread
Proportion p	$\hat{p} \pm z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$	$n\hat{p} \geq 10,\ n(1-\hat{p}) \geq 10$

Quick Review: Critical Values

Z Critical Values (for σ known or proportion):

Confidence Level	Two-sided $\alpha/2$	Two-sided $z_{\alpha/2}$	One-sided α	One-sided z_{α}
90%	0.05	1.645	0.10	1.282
95%	0.025	1.960	0.05	1.645
99%	0.005	2.576	0.01	2.326

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T and Chi-Square: Use tables with $\text{df} = n - 1$

A petroleum engineer measures crude oil viscosity. A random sample of $n = 64$ measurements yields $\bar{x} = 120$ cP. The population standard deviation is known to be $\sigma = 16$ cP. Construct a 95% CI for the true mean viscosity.

Problem 1: CI for Mean (σ known)

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What type of CI is this? What formula do we use?

What are we given?

Given: $n = 64$, $\bar{x} = 120$ cP, $\sigma = 16$ cP, 95% confidence

Problem 1: Solution

- **Step 1: Standard Error**

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{16}{\sqrt{64}} = \frac{16}{8} = 2$$

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- **Step 3: Margin of Error**

$$E = z_{\alpha/2} \times SE = 1.96 \times 2 = 3.92$$

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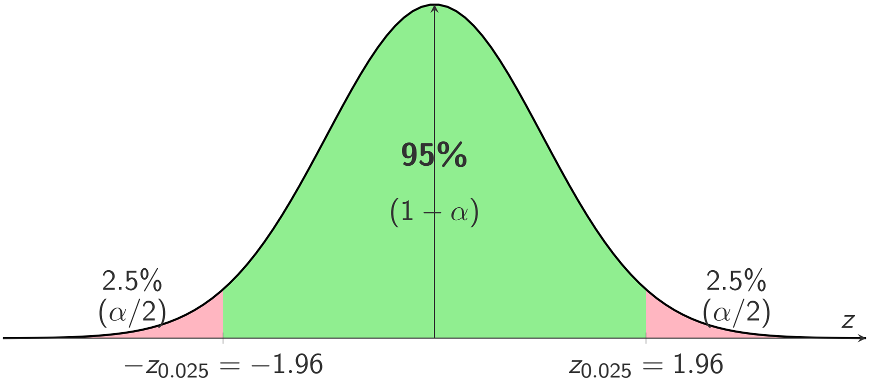
- **Step 4: Confidence Interval**

$$\bar{x} \pm E = 120 \pm 3.92 = \boxed{(116.08, 123.92)\ \text{cP}}$$

Problem 1: Illustration – Standard Normal Distribution



$Z \sim N(0,1)$ — CI: $\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} = 120 \pm 1.96 \times 2 = (116.08, 123.92)$

A robotics engineer measures positioning error during calibration. A sample of $n = 16$ measurements yields $\bar{x} = 85$ mm and sample standard deviation $s = 20$ mm. Construct a 95% CI for the true mean error.

Problem 2: CI for Mean (σ unknown)

A robotics engineer measures positioning error during calibration. A sample of $n = 16$ measurements yields $\bar{x} = 85$ mm and sample standard deviation $s = 20$ mm. Construct a 95% CI for the true mean error.

Problem 2: CI for Mean (σ unknown)

Since σ is unknown, which distribution do we use?

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Problem 2: CI for Mean (σ unknown)

Since σ is unknown, which distribution do we use?

What are we given?

Given: $n = 16$, $\bar{x} = 85$ mm, $s = 20$ mm, 95% confidence

- **Step 1: Standard Error** (using s instead of σ)

$$SE = \frac{s}{\sqrt{n}} = \frac{20}{\sqrt{16}} = \frac{20}{4} = 5$$

Problem 2: Solution

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$$SE = \frac{s}{\sqrt{n}} = \frac{20}{\sqrt{16}} = \frac{20}{4} = 5$$

- **Step 2: Critical Value** (t-distribution with $\mathrm{df} = n - 1 = 15$)

$$t_{\alpha/2,n-1} = t_{0.025,15} = 2.131$$

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- **Step 3: Margin of Error**

$$E = t_{\alpha/2,n-1} \times SE = 2.131 \times 5 = 10.655$$

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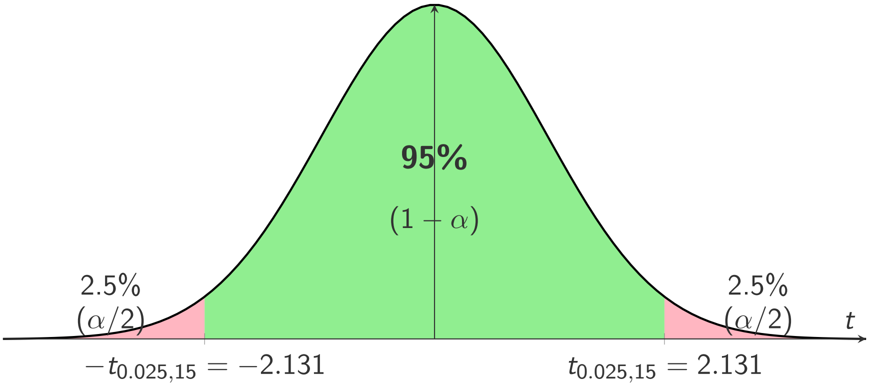
- **Step 4: Confidence Interval**

$$\bar{x} \pm E = 85 \pm 10.655 = \boxed{(74.35, 95.66) \text{ mm}}$$

Problem 2: Illustration – t-Distribution (df = 15)



t_{15} distribution — CI: $\bar{x} \pm t_{\alpha/2,n-1} \cdot \frac{s}{\sqrt{n}} = 85 \pm 2.131 \times 5 = (74.35, 95.66)$

An AI safety team tests an LLM for hallucinations. Out of $n = 400$ prompts, 60 produced hallucinated responses. Construct a 95% CI for the true hallucination rate.

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What conditions do we need to check first?

What are we given?

Given: $n = 400$, $x = 60$ hallucinations, 95% confidence

Problem 3: Solution

- **Step 1: Calc Proportion and Check ($n\hat{p} \geq 10$ ✓, $n(1-\hat{p}) \geq 10$ ✓) $\rightarrow z$**

$$\hat{p} = \frac{x}{n} = \frac{60}{400} = 0.15$$

Problem 3: Solution

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- **Step 2: Standard Error**

$$SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{0.15 \times 0.85}{400}} = \sqrt{0.000319} = 0.0179$$

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- **Step 3: Margin of Error** ($z_{0.025} = 1.96$)

$$E = 1.96 \times 0.0179 = 0.035$$

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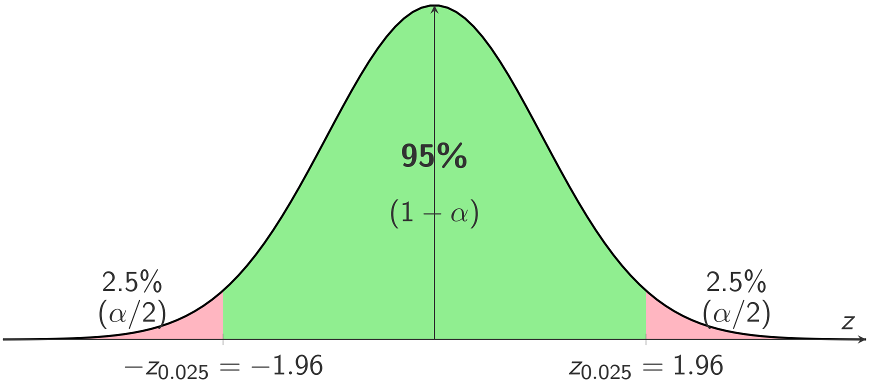
- **Step 4: Confidence Interval**

$$\hat{p} \pm E = 0.15 \pm 0.035 = \boxed{(0.115, 0.185) \text{ or } (11.5\%, 18.5\%)}$$

Problem 3: Illustration – Normal Approximation for Proportion



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An AI safety team measures LLM response time consistency. A sample of $n = 20$ queries yields sample variance $s^2 = 144$ ms^2. Construct a 95% CI for the true variance σ^2.

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Given: $n = 20$, $s^2 = 144$ ms^2, 95% confidence

- **Step 1: Degrees of Freedom**

$$df = n - 1 = 19$$

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$$\chi^2_{0.025,19} = 32.852 \quad \text{and} \quad \chi^2_{0.975,19} = 8.907$$

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- **Step 3: Compute** $(n-1)s^2$

$$(n-1)s^2 = 19 \times 144 = 2736$$

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$$\left[\frac{(n-1)s^2}{\chi^2_{\alpha/2}}, \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}}\right] = \left[\frac{2736}{32.852}, \frac{2736}{8.907}\right] = \boxed{(83.27, 307.18)\ \text{ms}^2}$$

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A quality engineer wants to estimate the mean tensile strength of an alloy. From past experience, $\sigma = 8$ MPa. How large a sample is needed so the error is within 2 MPa with 95% confidence?

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Problem 5: Sample Size Determination

What are we given?

Given: $\sigma = 8$ MPa, desired $E = 2$ MPa, 95% confidence

Problem 5: Solution

- **Recall:** Margin of error formula

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

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Problem 5: Solution

- **Recall:** Margin of error formula

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

- **Solve for n:** $n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E}\right)^2$
- **Substitute values:** $(z_{0.025} = 1.96)$

$$n = \left(\frac{1.96 \times 8}{2}\right)^2 = \left(\frac{15.68}{2}\right)^2 = (7.84)^2 = 61.47$$

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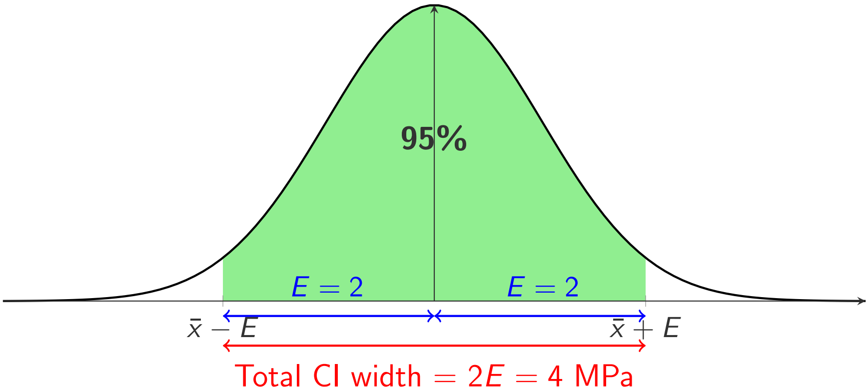
- **Round UP:** (always round up for sample size!)

$$n = \boxed{62 \text{ samples}}$$

Problem 5: Illustration – Margin of Error and Sample Size



$$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E}\right)^2 = \left(\frac{1.96 \times 8}{2}\right)^2 = 61.47 \to \boxed{62} \text{ (round UP!)}$$

A drilling engineer collects $n = 25$ measurements of drill bit wear with $\bar{x} = 2.4$ mm and $s = 0.5$ mm. Find a 95% **upper confidence bound** on the true mean wear.

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For one-sided bounds, what changes in our formula?

What are we given?

Given: $n = 25$, $\bar{x} = 2.4$ mm, $s = 0.5$ mm, 95% upper bound

Problem 6: Solution

- **Step 1: Standard Error**

$$SE = \frac{s}{\sqrt{n}} = \frac{0.5}{\sqrt{25}} = \frac{0.5}{5} = 0.1$$

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- **Step 2: Critical Value** (one-sided, df = 24)

$$t_{\alpha,n-1} = t_{0.05,24} = 1.711$$

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- **Step 3: Upper Confidence Bound**

$$\mu \leq \bar{x} + t_\alpha \cdot SE = 2.4 + 1.711 \times 0.1 = 2.4 + 0.171 = \boxed{2.571 \text{ mm}}$$

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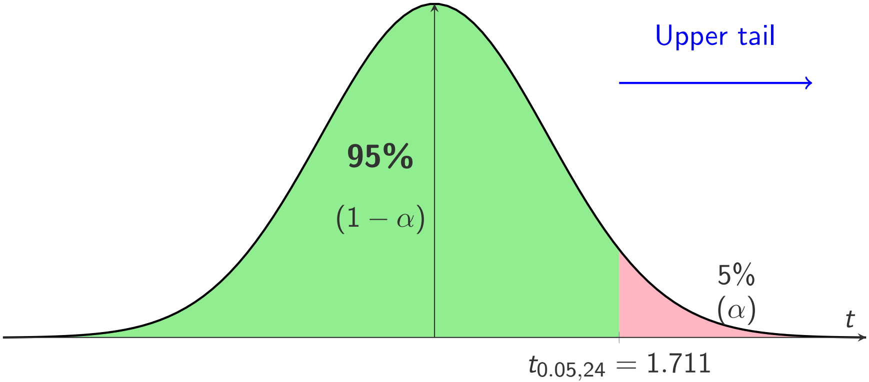
$$\mu \leq \bar{x} + t_{\alpha} \cdot SE = 2.4 + 1.711 \times 0.1 = 2.4 + 0.171 = \boxed{2.571 \text{ mm}}$$

Interpretation: We are 95% confident that the true mean is **at most** 2.571 mm.

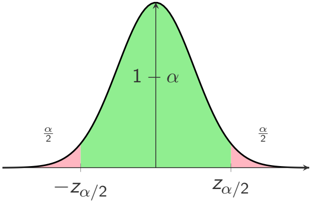
Problem 6: Illustration – One-Sided Upper Bound (t-distribution)



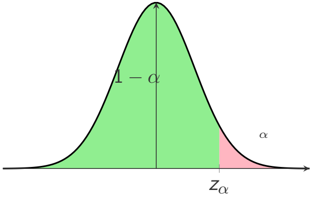
Upper bound: $\mu \leq \bar{x} + t_{\alpha} \cdot SE = 2.4 + 1.711 \times 0.1 = 2.571$ mm

Comparison: Two-Sided vs One-Sided Confidence Intervals

Two-Sided CI



One-Sided Upper Bound



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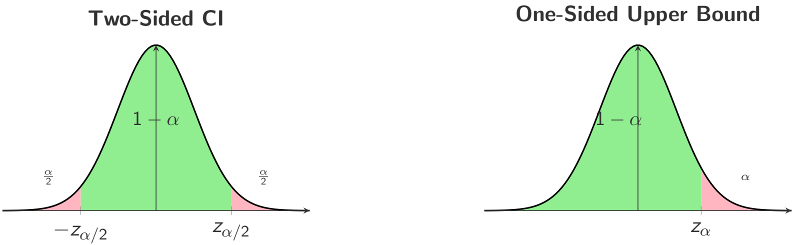

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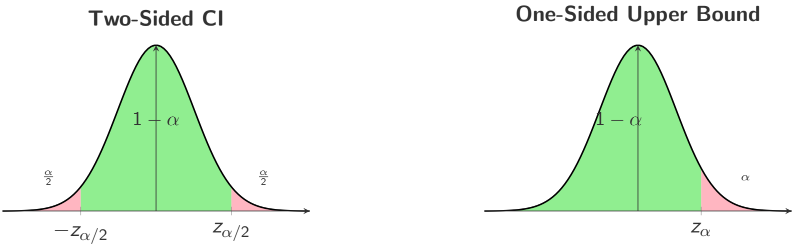

- **Two-sided:** Use $z_{\alpha/2}$ or $t_{\alpha/2}$ (split α into both tails)
- **One-sided:** Use z_α or t_α (all α in one tail)

Quick Decision Guide: Which CI Formula?

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  - Variance σ^2 $\Rightarrow$ χ^2**-distribution** (use $\chi^2_{\alpha/2,n-1}$ and $\chi^2_{1-\alpha/2,n-1}$)

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  - Variance σ^2 $\Rightarrow$ χ^2**-distribution** (use $\chi^2_{\alpha/2,n-1}$ and $\chi^2_{1-\alpha/2,n-1}$)
  - Proportion p $\Rightarrow$ **z-distribution** with $SE = \sqrt{\hat{p}(1-\hat{p})/n}$
- **Two-sided or one-sided?**
  - Two-sided: Use $z_{\alpha/2}$ or $t_{\alpha/2}$ (split α into **both** tails)
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