ISE 315: Engineering Statistics

Lecture 8: Introduction to Hypothesis Testing

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Based on Montgomery & Runger, Applied Statistics and Probability for Engineers, 6th Ed.

Lecture 8

Introduction to Hypothesis Testing (Chapter 9)

Lecture 8 Outline

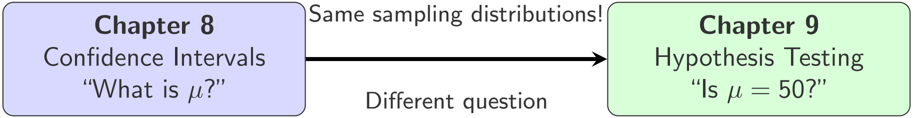
- Introduction to Hypothesis Testing (Chapter 9)
  - From confidence intervals to hypothesis tests
  - Statistical hypotheses: H_0 vs H_1
  - Type I and Type II errors
  - Test statistics and critical regions
- Quiz #2 (30 minutes)

Chapter 9

Tests of Hypotheses for a Single Sample

From Confidence Intervals to Hypothesis Testing

The Big Picture



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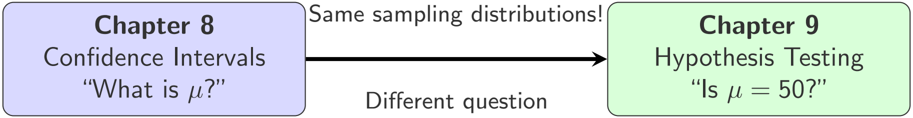

- **Chapter 8 (CI):** Estimate an unknown parameter
  - "The true mean viscosity is between 116.08 and 123.92 cP"

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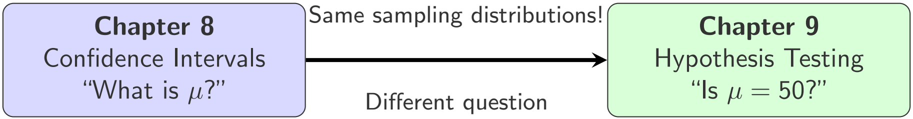

- **Chapter 8 (CI):** Estimate an unknown parameter
  - “The true mean viscosity is between 116.08 and 123.92 cP”
- **Chapter 9 (HT):** Test a claim about a parameter
  - “Is the true mean viscosity equal to 120 cP?”

Motivating Example: Crude Oil Viscosity

Scenario: A petroleum engineer measures crude oil viscosity.

- Sample: $n = 64$ measurements, $\bar{x} = 120$ cP, known $\sigma = 16$ cP

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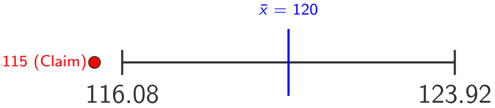

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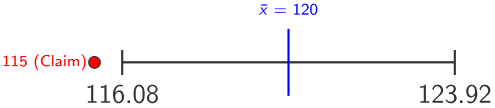
Chapter 9 Question: The supplier claims the mean viscosity is 115 cP. Is this claim supported by our data?

- 115 is **outside** our 95% CI! (the claim might be false?)



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Key Idea: We use probability to quantify the evidence against the hypothesis.

The Two Hypotheses

H_0 and H_1

Null Hypothesis H_0:

- The “default” or “status quo”
- What we assume is true
- Usually contains “$=$”
- Example: $H_0 : \mu = 115$

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Important: We never "accept" H_0.
We either **reject** H_0 or **fail to reject** H_0.

Three Types of Hypothesis Tests

Type	**Hypotheses**	**Example**
Two-tailed	$H_0 : \mu = \mu_0$ vs $H_1 : \mu \neq \mu_0$	Is μ different from 115?

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Can you guess the connection to one- or two-sided CIs?

Type I and Type II Errors

Two Ways to Be Wrong

	Truth (Unknown)	
Decision	H_0 is True	H_0 is False

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- **Type II Error (β):** Failing to reject H_0 when it's actually false
  - "False negative" or "missed detection", probability $= \beta$

Engineering Analogy: Quality Control

Type I vs Type II Errors

Scenario: Testing if a batch of parts meets specifications.

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Trade-off: Reducing α typically increases β (and vice versa).

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You have seen this in Chapter 8: $\alpha = 0.05$ for hypothesis test $\leftrightarrow$ 95% CI

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For testing μ when σ is known: use the Z-test statistic: $Z_0 = \frac{\bar{X}-\mu_0}{\sigma/\sqrt{n}}$

- $\bar{X}$ = sample mean (what we observed)
- μ_0 = hypothesized value (from H_0)
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- $\sigma / \sqrt{n}$ = standard error

Interpretation: Z_0 tells us how many standard errors $\bar{X}$ is from μ_0.

Example: Testing the Supplier's Claim

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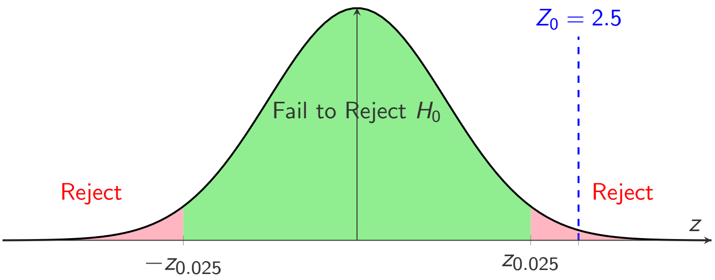
Test statistic: $Z_0 = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}} = \frac{120-115}{16/\sqrt{64}} = \frac{5}{2} = 2.5$
Question: Is $Z_0 = 2.5$ large enough to reject H_0?

We need a **decision rule**... (next slide!)

Critical Region and Decision Rule



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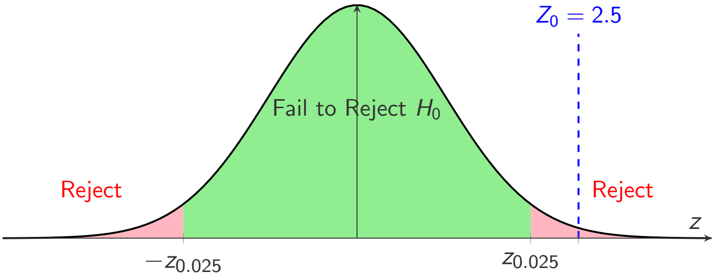

Decision Rule (for $\alpha = 0.05$, two-tailed): Reject H_0 if $|Z_0| > z_{0.025} = 1.96$

- Since $|2.5| > 1.96$, we **reject** H_0

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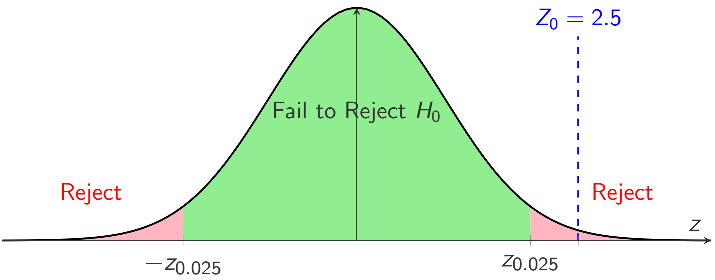

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Conclusion: The data provides sufficient evidence that the claim is **false**.

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Next lecture: More examples, P-values, and tests for σ unknown

Coming Up in Chapter 9...

Parameter	Test Statistic
Mean μ (σ known)	$Z_0 = \frac{\bar{X}-\mu_0}{\sigma/\sqrt{n}}$
Mean μ (σ unknown)	$T_0 = \frac{\bar{X}-\mu_0}{S/\sqrt{n}}$
Variance σ^2	$\chi_0^2 = \frac{(n-1)S^2}{\sigma_0^2}$
Proportion p	$Z_0 = \frac{\hat{p}-p_0}{\sqrt{p_0(1-p_0)/n}}$

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Notice: Same distributions as confidence intervals!