ISE 315: Engineering Statistics

Lecture 9: Hypothesis Testing for a Single Sample

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Based on Montgomery & Runger, Applied Statistics and Probability for Engineers, 6th Ed.

Lecture 9

Hypothesis Testing for a Single Sample (Chapter 9, cont'd)

Lecture 9 Outline

- Announcements:
  - HW #3 due next week
  - Major Exam 1 in Week 8 (Chapters 7, 8, 9, 10)
- Review: Key concepts from Lecture 8
- P-values in Hypothesis Tests
- General Procedure for Hypothesis Tests
- Tests on the Mean, σ Known

Quick Review

What We Learned in Lecture 8

Lecture 8 Recap: The Big Ideas

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Today: We add P-values, more examples, and tests when σ is unknown.

Section 9-1.4

P-Values in Hypothesis Tests

Two Approaches to Making a Decision

Approach 1: Critical Value (Fixed Significance Level)

- Choose α beforehand (e.g., $\alpha = 0.05$)
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- Compare test statistic to critical value
- Decision: Reject or fail to reject

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Approach 2: P-Value

- Compute the probability of observing a test statistic *at least as extreme* as the one calculated, assuming H_0 is true
- Compare P-value to α
- Both approaches give the **same conclusion**!

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$$\text{If P-value} > \alpha \Rightarrow \textbf{Fail to reject } H_0$$

Intuition: A small P-value means the data is very unlikely under H_0, so reject H_0.

Computing P-Values for the Z-Test Z_0

Test Type	**Hypotheses**	**P-value**
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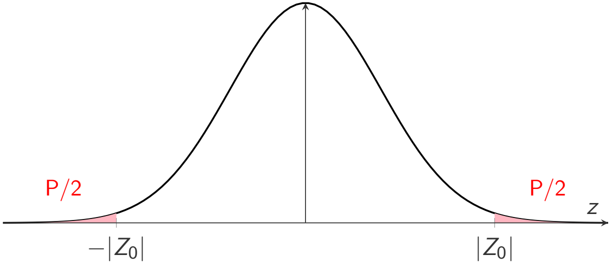
Recall: $\Phi(z) = P(Z \leq z)$ is the standard Normal CDF (the Z-table!).

P-Value Illustrated: Two-Tailed Test



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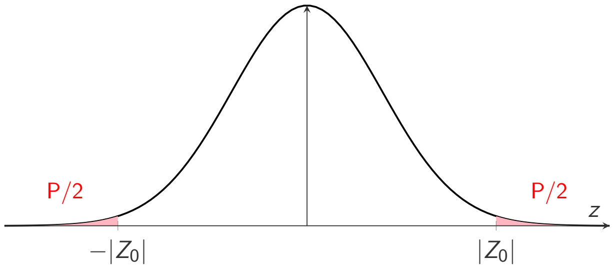

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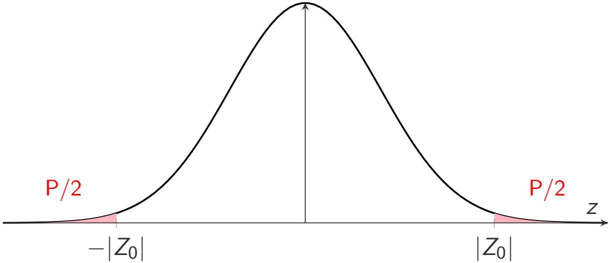

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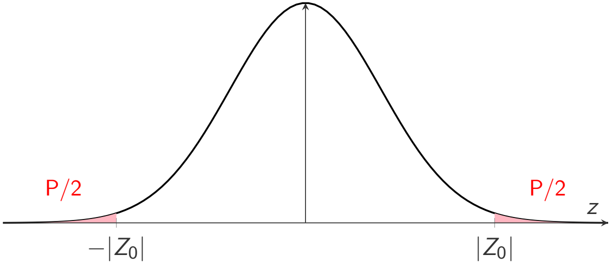

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Revisiting the supplier claim with a P-value

Example: Crude Oil Viscosity

Recall: $n = 64$, $\bar{x} = 120$ cP, $\sigma = 16$ cP, $H_0 : \mu = 115$, $H_1 : \mu \neq 115$

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The P-value tells us the **strength of evidence <u>against</u>** H_0:

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P-value	**Evidence against H_0**
> 0.10	Little or no evidence
0.05 to 0.10	Weak evidence
0.01 to 0.05	Moderate evidence
0.001 to 0.01	Strong evidence
< 0.001	Very strong evidence

Section 9-1.6

General Procedure for Hypothesis Tests

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Always state conclusion in context of the problem

"We reject the null hypothesis and claim that there is sufficient evidence at the 5% significance level that the true mean viscosity is not 115 cP."

Variations of Tests

Z-test for the mean (σ known),
t-test for the mean (σ unknown),
χ^2-test for the variance (σ^2)

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Assumption: Population is Normal, or CLT applies (what conditions?)

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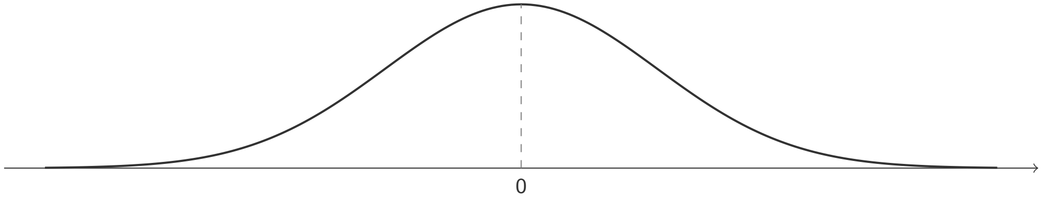

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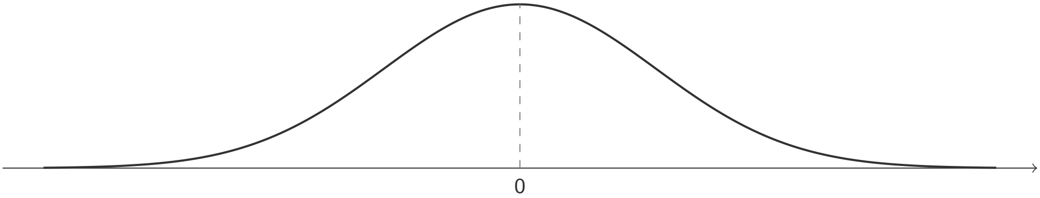

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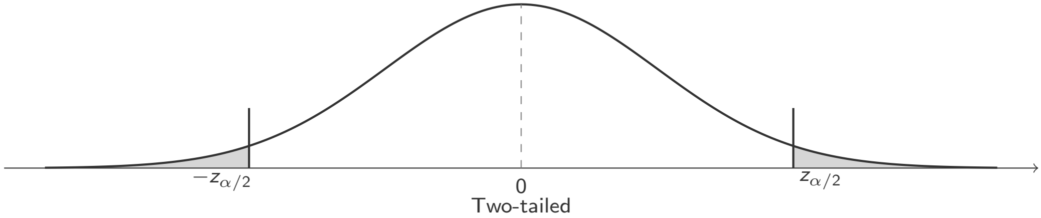

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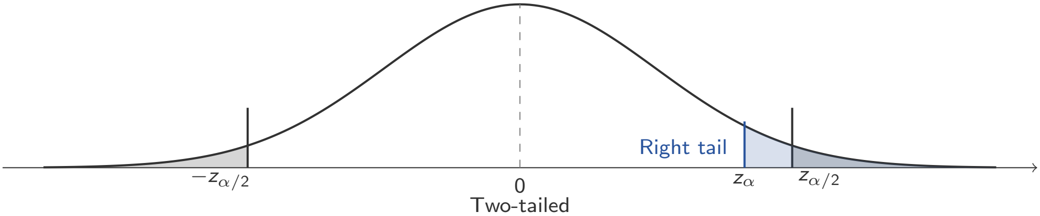

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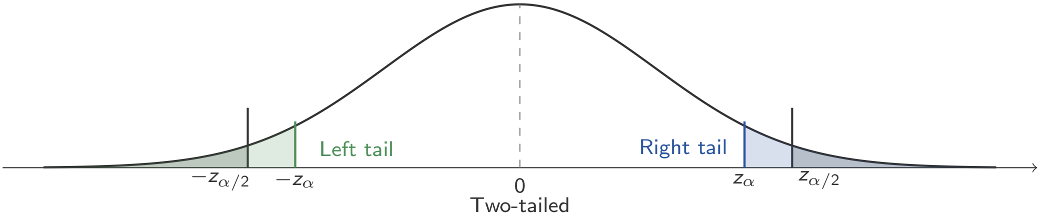

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Conclusion: Sufficient evidence that the mean bit life exceeds 200 hours.

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Decision rule: Reject H_0 if $|t_0| > t_{1-\alpha/2,\, n-1}$ (two-tailed)

A soil laboratory claims that a new pile-driving technique yields an average intrusion $> 12.5\,\text{m}$. Data: $n = 9$, $\bar{x} = 13.2\,\text{m}$, $s = 1.5\,\text{m}$. Test at $\alpha = 0.05$.

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Conclusion: Not enough evidence to support the claim that the mean intrusion depth exceeds 12.5 meters.

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