ISE 315: Engineering Statistics

Supplementary Material: Finding P-Value from the Z-Table

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Supplementary Material

Finding P-Value from the Z-Table (Standard Normal)

P-Value and Its Bounds from the Z-Table?

When σ is known and the sample size is large, we use the Z-**test**. Software gives exact p-values, but on exams and quizzes you work with the **standard normal table**.

Unlike the t, χ^2, and F tables (which list critical values for specific α levels), the standard normal table gives cumulative probabilities $\Phi(z) = P(Z \leq z)$.

Two common table formats:

- **Cumulative $\Phi(z)$ table** (Appendix Table III): gives $P(Z \leq z)$ directly
- **Critical value table**: lists z_α values for selected α levels

The Z-table often lets you find **exact** (or near-exact) p-values, not just bounds. But when your $|z_0|$ falls between tabulated values, you still report bounds.

How the Z-Table Is Organized

The cumulative standard normal table (Appendix Table III):

- Rows $=$ z-value to one decimal place (e.g., 1.9)
- Columns $=$ second decimal place (e.g., 0.06)
- Entry $= \Phi(z) = P(Z \leq z)$

Example: Finding $\Phi(1.96)$

z	$\cdots$	0.05	**0.06**	0.07	$\cdots$
$\vdots$					
1.9	$\cdots$	0.9744	**0.9750**	0.9756	$\cdots$
$\vdots$					

The General Strategy for the Z-Table

Step 1: Compute the test statistic z_0

- For mean: $z_0 = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$
- For proportion: $z_0 = \dfrac{\hat{p} - p_0}{\sqrt{p_0(1 - p_0)/n}}$

Step 2: Find the upper-tail area using $|z_0|$

- Look up $\Phi(|z_0|)$ from the table
- Upper-tail area $= 1 - \Phi(|z_0|)$

Step 3: Convert to p-value

- **One-tailed:** $p = 1 - \Phi(|z_0|)$
- **Two-tailed:** $p = 2\left[1 - \Phi(|z_0|)\right]$

When $|z_0|$ Falls Between Table Entries

Most standard normal tables go to **two decimal places**. If your test statistic has more precision, you may need to **bound** the p-value.

Method: If $z_a < |z_0| < z_b$ where z_a and z_b are consecutive table entries:

- $\Phi(z_a) < \Phi(|z_0|) < \Phi(z_b) \quad$ (since Φ is increasing)
- $1 - \Phi(z_b) < 1 - \Phi(|z_0|) < 1 - \Phi(z_a)$

For a one-tailed test:

$$1 - \Phi(z_b) < p\text{-value} < 1 - \Phi(z_a)$$

For a two-tailed test:

$$2[1 - \Phi(z_b)] < p\text{-value} < 2[1 - \Phi(z_a)]$$

Common Z Critical Values as Benchmarks

These values are worth memorizing for quick p-value estimation:

z_α	**Upper-tail area α**	**Two-tailed p-value**
1.282	0.10	0.20
1.645	0.05	0.10
1.960	0.025	0.05
2.326	0.01	0.02
2.576	0.005	0.01
3.090	0.001	0.002

Z-Table Example 1

A refinery's historical data shows the sulfur content in a crude oil blend has $\sigma = 0.5\%$ (known). A sample of $n = 40$ batches gives $\bar{x} = 2.68\%$. Test whether the mean sulfur content differs from the target of $\mu_0 = 2.50\%$ at $\alpha = 0.05$. Estimate the p-value.

Hypotheses: $H_0 : \mu = 2.50$ vs $H_1 : \mu \neq 2.50$. Two-tailed Z-test.

Test statistic: $z_0 = \dfrac{2.68 - 2.50}{0.5/\sqrt{40}} = \dfrac{0.18}{0.0791} = 2.28$

P-value: From table: $\Phi(2.28) = 0.9887$.
Upper tail: $1 - 0.9887 = 0.0113$. Two-tailed: $p = 2(0.0113) = \boxed{0.0226}$

Decision: Since $p = 0.0226 < 0.05 = \alpha$, we **reject** H_0. The mean sulfur content differs significantly from the 2.50% target.

Z-Table Example 2

A pipeline manufacturer claims that at least 95% of welds pass X-ray inspection. In a random sample of $n = 200$ welds, 183 pass inspection ($\hat{p} = 0.915$). Test whether the pass rate is below 95% at $\alpha = 0.05$. Estimate the p-value.

Hypotheses: $H_0 : p = 0.95$ vs $H_1 : p < 0.95$. One-tailed lower Z-test for proportions.

Test statistic: $z_0 = \dfrac{0.915 - 0.95}{\sqrt{0.95 \times 0.05/200}} = \dfrac{-0.035}{0.01541} = -2.27$

P-value: Since this is a lower-tail test, $p = P(Z < -2.27) = \Phi(-2.27)$.
From table: $\Phi(-2.27) = 1 - \Phi(2.27) = 1 - 0.9884 = \boxed{0.0116}$

Decision: Since $p = 0.0116 < 0.05 = \alpha$, we **reject** H_0. The data provides significant evidence that the weld pass rate is below 95%.

Common Mistakes to Avoid

- **Forgetting to multiply by 2 for two-tailed tests.**
  The table tail area is one-sided. For a two-tailed p-value, always double it.
- **Using $\Phi(z_0)$ directly as the p-value for an upper-tail test.**
  Upper tail area $= 1 - \Phi(z_0)$, *not* $\Phi(z_0)$ itself!
- **Sign confusion with lower-tail tests.**
  If $z_0 < 0$, use the symmetry property: $\Phi(-z) = 1 - \Phi(z)$. The p-value for a lower-tail test is $P(Z < z_0) = \Phi(z_0)$.
- **Using the Z-table when σ is unknown and n is small.**
  If σ is unknown and $n < 30$, use the t-table instead. The Z-table requires known σ or large n.

Summary

- The Z-**table** gives cumulative probabilities $\Phi(z)$, not critical values at fixed α levels.
- For **upper-tail** tests, compute $p = 1 - \Phi(|z_0|)$.
- For **lower-tail** tests, compute $p = \Phi(z_0) = 1 - \Phi(|z_0|)$.
- For **two-tailed** tests, double the one-tail area: $p = 2[1 - \Phi(|z_0|)]$.
- The Z-table typically gives **exact or near-exact** p-values, unlike the $t/\chi^2/F$ tables which usually give bounds.
- Always state your final answer clearly: exact p-value or bound.

Memorize the key benchmarks ($z = 1.645, 1.96, 2.326, 2.576$) for quick p-value estimation on exams!