ISE 315: Engineering Statistics

Lecture 5: Statistical Intervals for a Single Sample (Part 1)

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Based on Montgomery & Runger, Applied Statistics and Probability for Engineers, 6th Ed.

Lecture 5

Statistical Intervals for a Single Sample (Part 1)

Lecture 5 Outline (Textbook Chapter 3)

- Announcements:
  - HW 1 due Wednesday before class
  - HW 2 due next Wednesday before class
  - Major Exam 1 in Week 8 (Textbook Chapters 2–5)
- From Point Estimates to Interval Estimates
- Confidence Interval on the Mean (σ^2 Known)
- Choice of Sample Size
- One-Sided Confidence Bounds

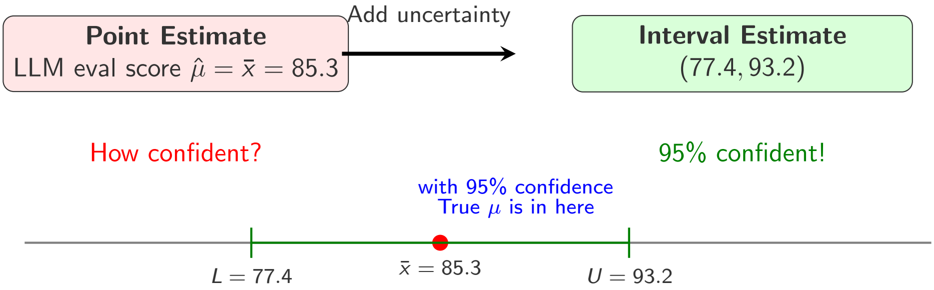
Quick survey!

Live poll

How confident do you feel about point estimation from Chapter 2?

Visit **pe.app/ise315** to answer!

Chapter 3 Concept Review



Textbook: Chapter 3, Statistical Intervals for a Single Sample ((3.1) onward)

Key Terminologies

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4. **Confidence Level:**
   - $1 - \alpha =$ probability the method produces an interval containing θ
   - 95% CI $\Rightarrow \alpha = 0.05$

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The margin of error combines critical value and standard error: $E = z_{\alpha/2} \times SE$

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One-sided intervals use z_α instead of $z_{\alpha/2}$ (all α on one side)

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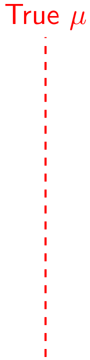
  - "The interval (L, U) was constructed using a method that captures μ 95% of the time"

Visualizing Confidence Intervals: Repeated Sampling



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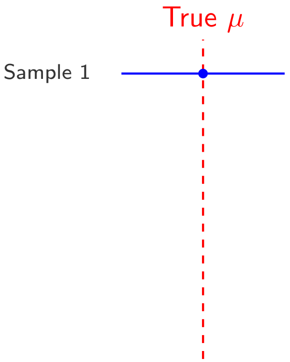

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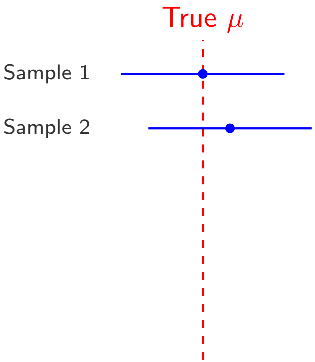

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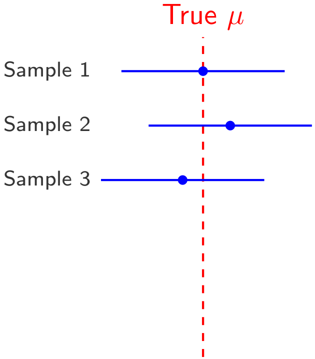

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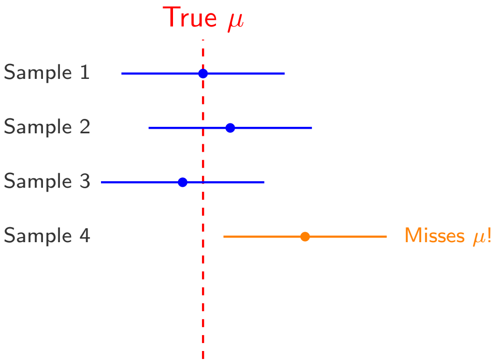

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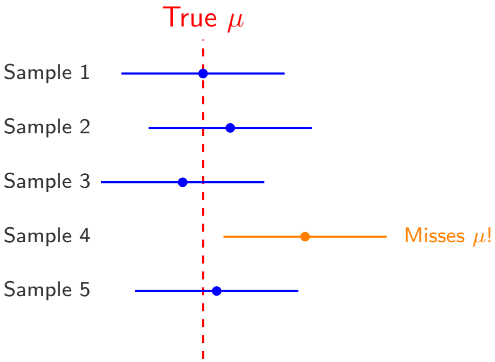

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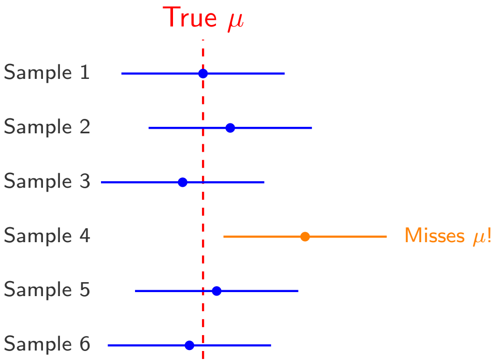

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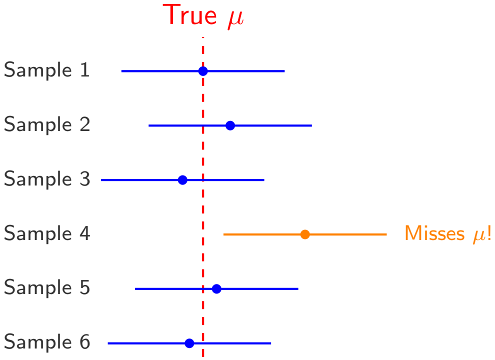

With 95% CI: about 95 out of 100 intervals capture μ

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Two-Sided Confidence Interval (σ Known)

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These are equations 3.1 and 3.3 in the textbook — the full recipe is Procedure 3.1.

One-Sided Bounds and Sample Size (σ Known)

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$$\mu \geq \bar{x} - z_\alpha \frac{\sigma}{\sqrt{n}} \quad \text{(lower)}, \qquad \mu \leq \bar{x} + z_\alpha \frac{\sigma}{\sqrt{n}} \quad \text{(upper)} \tag{3.2}$$

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Note: one-sided bounds use z_{α}, not $z_{\alpha/2}$.

Common Critical Values $z_{\alpha/2}$ (Table 3.1 in the textbook)

Confidence Level	α	$\alpha/2$	$z_{\alpha/2}$
90%	0.10	0.05	1.645
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- **Memory tip:** For 95% CI, use $z \approx 2$

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- Sample size formula (3.4): $n = \left(\frac{z_{\alpha/2}\sigma}{E}\right)^2$

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- Compare confidence intervals with different confidence levels or sample sizes

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- Construct interval: $\bar{x} \pm E$ or $\bar{x} - E$ (lower bound) or $\bar{x} + E$ (upper bound)

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Same approach:
Parameter $\rightarrow$ Sampling distribution $\rightarrow$ Get critical value $\rightarrow$ CI

A logistics analyst measures container dwell time at a port. A random sample of $n = 36$ containers gives sample mean $\bar{x} = 3.750$ days. Historical data shows $\sigma = 0.90$ days. Construct a 95% CI for the true mean dwell time.

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Plan: Two-sided z-interval, (3.1) via Procedure 3.1

Step 1: Calculate standard error

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We are 95% confident that the true mean dwell time μ is between 3.456 and 4.044 days

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$$= \left(\frac{23.52}{3}\right)^2 = (7.84)^2 = 61.47 \approx 62 \text{ users (round up!)}$$

An AV safety team measures the pedestrian-detection distance of a perception model in $n = 16$ test runs. Sample mean $\bar{x} = 185.0$ m, known $\sigma = 10$ m. Find a 95% lower confidence bound for the true mean detection distance.

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Interpretation: We are 95% confident that the model detects pedestrians at *at*

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