ISE 315: Engineering Statistics

Lecture 13: Major Exam 1 Review (Chapters 3, 4, 5)

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Based on Montgomery & Runger, Applied Statistics and Probability for Engineers, 6th Ed.

Lecture 13

Major Exam 1 Review: Chapters 3, 4, and 5

Lecture 13 Outline

- **Chapter 3: Statistical Intervals for a Single Sample** (Lectures 5–7)
  - CI on μ (σ known) (3.1), one-sided bounds (3.2), sample size (3.4)
  - CI on μ (σ unknown, t) (3.5) / CI on σ^2 (3.6)
  - CI on p (3.7) and sample size (3.8)–(3.9)
  - Prediction (3.10) and tolerance (3.11) intervals

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  - Prediction (3.10) and tolerance (3.11) intervals
- **Chapter 4: Hypothesis Testing for a Single Sample** (Lectures 8–10)
  - Framework, P-values, seven steps (Procedure 4.2), CI–HT connection
  - Z-test (4.3), β and power (4.10), sample size (4.11)
  - t-test (4.6) / χ^2-test (4.7) / proportion Z-test (4.8)

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  - t-test (4.6) / χ^2-test (4.7) / proportion Z-test (4.8)
- **Chapter 5: Inference for Two Samples** (Lectures 11–12)
  - Two-sample Z (5.1) / pooled t (5.4) / Welch's t (5.6)
  - Paired t (5.8) / F-test (5.13) / two proportions (5.10)

Chapter 3

Statistical Intervals for a Single Sample

(Lectures 5, 6, 7)

CI on the Mean μ (σ^2 Known)

Procedure 3.1 | Lecture 5

Two-Sided $100(1-\alpha)\%$ **Confidence Interval:**

$$\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \le \mu \le \bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \tag{3.1}$$

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One-Sided Confidence Bounds:

$$\text{Lower:}\quad \mu \geq \bar{x} - z_{\alpha}\,\frac{\sigma}{\sqrt{n}} \qquad\qquad \text{Upper:}\quad \mu \leq \bar{x} + z_{\alpha}\,\frac{\sigma}{\sqrt{n}} \tag{3.2}$$

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Margin of Error (3.3) and Sample Size (3.4):

$$E = z_{\alpha/2}\,\frac{\sigma}{\sqrt{n}} \qquad \Longrightarrow \qquad n = \left\lceil \left(\frac{z_{\alpha/2}\,\sigma}{E}\right)^2 \right\rceil \quad \text{(always round } \textbf{up}\text{)}$$

CI on the Mean μ (σ^2 Unknown)

Procedure 3.2 | Lecture 6

Replace σ with s and z with t:

$$\bar{x} - t_{\alpha/2,\, n-1} \frac{s}{\sqrt{n}} \;\leq\; \mu \;\leq\; \bar{x} + t_{\alpha/2,\, n-1} \frac{s}{\sqrt{n}} \tag{3.5}$$

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Key facts about the t-distribution:

- Symmetric about 0, heavier tails than the standard normal
- Degrees of freedom: $\nu = n - 1$
- $t_{\alpha/2,\, n-1} > z_{\alpha/2}$ for all finite $n \Rightarrow t$-interval is **always wider**
- As df $\to \infty$, $t \to Z$

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Assumptions: Population approximately normal (critical for small n).
For large n (≥ 30), the CLT makes the t-interval robust to non-normality.

CI on the Variance σ^2 and Std Dev σ

Procedure 3.3 | Lecture 6

$100(1-\alpha)\%$ **CI for σ^2:**

$$\frac{(n-1)\,s^2}{\chi^2_{\alpha/2,\,n-1}} \le \sigma^2 \le \frac{(n-1)\,s^2}{\chi^2_{1-\alpha/2,\,n-1}} \tag{3.6}$$

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CI for σ: take the square root of both endpoints.

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CI for σ**:** take the square root of both endpoints.

Remember:

- Not symmetric about s^2; the **larger** χ^2 value builds the **lower** limit
- **Requires** a normal population — very sensitive to this assumption
- Critical values from Table B.4

CI on the Proportion p and Its Sample Size

Procedure 3.4 | Lecture 6

Point estimator $\hat{p} = x/n$; $100(1-\alpha)\%$ **CI for** p (needs $n\hat{p} \geq 5$, $n(1-\hat{p}) \geq 5$):

$$\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \tag{3.7}$$

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Sample size with a prior estimate $\hat{p}$**:**

$$n = \left\lceil \frac{z_{\alpha/2}^2 \, \hat{p}(1-\hat{p})}{E^2} \right\rceil \tag{3.8}$$

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Without a prior (conservative, $\hat{p} = 0.5$ maximizes $\hat{p}(1-\hat{p})$):

$$n = \left\lceil \frac{z_{\alpha/2}^2 \times 0.25}{E^2} \right\rceil \tag{3.9}$$

Prediction and Tolerance Intervals

Lecture 7

Prediction interval for ONE future observation:

$$\bar{x} \pm t_{\alpha/2,\,n-1}\, s\sqrt{1+\frac{1}{n}} \tag{3.10}$$

Wider than the CI (3.5): the "1+" carries the noise of the new observation itself.

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Tolerance interval covering a proportion of the population:

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with the factor k from Table 3.5 (depends on n, coverage, and confidence).

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Keep the three apart: CI → the *mean*; PI → the *next value*; TI → a *fraction of all values*.

Choosing the Right Interval

Procedure 3.5; summary in Table 3.7

Target	**Conditions**	**Interval**	**Book Eq.**
μ	σ known	z-interval	(3.1)

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First question on any interval problem: what exactly is the quantity the interval must capture?

Chapter 4

Hypothesis Testing for a Single Sample

(Lectures 8, 9, 10)

The Hypothesis Testing Framework

Procedure 4.2 | Lecture 8

Hypothesis pair (4.1): H_0 (with "$=$") vs H_1 (with $\neq$, $<$, or $>$). The seven steps:

1. Identify the parameter of interest
2. State H_0
3. State H_1 (fixes the tail direction)
4. Choose the test statistic
5. State the rejection criterion for α
6. Compute
7. Conclude **in context**

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Decision rule (Procedure 4.1):

If P-value $\leq \alpha \Rightarrow$ **Reject** H_0

If P-value $> \alpha \Rightarrow$ **Fail to reject** H_0

Type I Error, Type II Error, and Power

(4.2); see Table 4.2 | Lecture 8

	H_0 **is True**	H_0 **is False**
Reject H_0	Type I Error (α)	Correct (Power $= 1 - \beta$)
Fail to reject H_0	Correct ($1 - \alpha$)	Type II Error (β)

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Definitions (4.2):

- $\alpha = P(\text{Reject } H_0 \mid H_0 \text{ true})$ — false alarm
- $\beta = P(\text{Fail to reject } H_0 \mid H_0 \text{ false})$ — missed detection
- Power $= 1 - \beta$

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P-Values

(4.4) | Lecture 9

Definition (4.4): The smallest significance level at which we would reject H_0 given the data.

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Z**-test P-values** (4.5):

Test Type	**P-value**
Two-tailed ($H_1 : \mu \neq \mu_0$)	$2\left[1 - \Phi(\lvert Z_0 \rvert)\right]$
Right-tailed ($H_1 : \mu > \mu_0$)	$1 - \Phi(Z_0)$
Left-tailed ($H_1 : \mu < \mu_0$)	$\Phi(Z_0)$

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For t and χ^2 tests (Procedure 4.3): bound the P-value between two table columns,

Connection Between CI and HT

Lecture 8

Key equivalence for a two-tailed test at level α:

$$\boxed{\begin{array}{c}\text{Reject } H_0 : \theta = \theta_0 \text{ at level } \alpha \\ \iff \\ \theta_0 \text{ falls } \textbf{outside} \text{ the } 100(1-\alpha)\% \text{ CI for } \theta\end{array}}$$

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$$\iff$$
$$\theta_0 \text{ falls } \textbf{outside} \text{ the } 100(1-\alpha)\% \text{ CI for } \theta$$

- θ_0 **inside** the CI $\Rightarrow$ fail to reject $\Rightarrow$ P-value $> \alpha$
- θ_0 **outside** the CI $\Rightarrow$ reject $\Rightarrow$ P-value $< \alpha$
- Applies to μ, σ^2, p, $\mu_1 - \mu_2$, μ_D, σ_1^2/σ_2^2, and $p_1 - p_2$

Z-Test on the Mean (σ Known)

Lectures 8–9

Test statistic:

$$Z_0 = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \tag{4.3}$$

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Decision rules and P-values (4.5):

Alternative	**Reject H_0 if**	**P-value**
$H_1 : \mu \neq \mu_0$	$\|Z_0\| > z_{\alpha/2}$	$2\left[1 - \Phi(\|Z_0\|)\right]$
$H_1 : \mu > \mu_0$	$Z_0 > z_\alpha$	$1 - \Phi(Z_0)$
$H_1 : \mu < \mu_0$	$Z_0 < -z_\alpha$	$\Phi(Z_0)$

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$H_1 : \mu > \mu_0$	$Z_0 > z_\alpha$	$1 - \Phi(Z_0)$
$H_1 : \mu < \mu_0$	$Z_0 < -z_\alpha$	$\Phi(Z_0)$

Type II Error (β), Power, and Sample Size

Procedure 4.4 | Lecture 10

Computing β (Procedure 4.4):

1. Write the fail-to-reject region in $\bar{x}$ units — the acceptance region (4.9): e.g. one-sided, $\bar{x}_c = \mu_0 + z_\alpha \, \sigma/\sqrt{n}$
2. $\beta = P(\bar{X} \in \text{fail-to-reject region} \mid \mu = \mu_{\text{true}})$, standardizing under the truth

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Two-tailed formula (4.10): $\beta = \Phi\left(z_{\alpha/2} - \dfrac{|\delta|\sqrt{n}}{\sigma}\right) - \Phi\left(-z_{\alpha/2} - \dfrac{|\delta|\sqrt{n}}{\sigma}\right)$, $\delta = \mu_{\text{true}} - \mu_0$

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Sample size for target power (4.11):

Two-tailed: $n = \left\lceil \dfrac{(z_{\alpha/2} + z_\beta)^2\, \sigma^2}{\delta^2} \right\rceil$ One-tailed: replace $z_{\alpha/2}$ with z_α

t-Test on the Mean (σ Unknown)

Lecture 9

Test statistic:

$$T_0 = \frac{\bar{X} - \mu_0}{S/\sqrt{n}}, \quad \mathrm{df} = n - 1 \tag{4.6}$$

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$H_1 : \mu > \mu_0$	$T_0 > t_{\alpha,\, n-1}$
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When to use: σ unknown (almost always in practice). Population approximately

χ^2-Test on the Variance σ^2

Lecture 10

Test statistic:

$$\chi_0^2 = \frac{(n-1)\,S^2}{\sigma_0^2}, \quad \mathrm{df} = n - 1 \tag{4.7}$$

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$H_1 : \sigma^2 > \sigma_0^2$	$\chi_0^2 > \chi^2_{\alpha,\,n-1}$
$H_1 : \sigma^2 < \sigma_0^2$	$\chi_0^2 < \chi^2_{1-\alpha,\,n-1}$

Critical assumption: the population **must be normal** — this test is **very sensitive**

Z-Test on the Proportion p

Lecture 10

Test statistic:

$$Z_0 = \frac{\hat{p} - p_0}{\sqrt{p_0(1 - p_0)/n}} \tag{4.8}$$

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Important: the test uses p_0 (the **hypothesized** value) in the standard error; the CI (3.7) uses $\hat{p}$ instead.

Chapter 5

Statistical Inference for Two Samples

(Lectures 11, 12)

Two-Sample Z-Test and CI (σ_1^2, σ_2^2 Known)

Lecture 11

$$Z_0 = \frac{(\bar{X}_1 - \bar{X}_2) - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \tag{5.1}$$

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Two-Sample Z-Test and CI (σ_1^2, σ_2^2 Known)

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Decision rules: same structure as any Z-test. Δ_0 is the hypothesized difference (usually 0).

Pooled t-Test ($\sigma_1^2 = \sigma_2^2$ Assumed, Both Unknown)

Lecture 11

Pooled variance (weighted average of the sample variances):

$$S_p^2 = \frac{(n_1 - 1)\,S_1^2 + (n_2 - 1)\,S_2^2}{n_1 + n_2 - 2} \tag{5.3}$$

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Procedure 5.1 | Lecture 12

Test statistic (put the **larger** variance on top so $F_0 \geq 1$):

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Key distinction: HT uses **pooled** $\hat{p}$ — CI uses **individual** $\hat{p}_1, \hat{p}_2$

Summary

Master Table, Key Relationships, and Common Pitfalls

Master Table: Which Test / CI to Use

The book's version is Table 5.6 — bookmark it

Parameter	Conditions	Test Statistic	Test Eq.	CI Eq.
μ	σ known	$Z_0 = (\bar{X} - \mu_0)/(\sigma/\sqrt{n})$	(4.3)	(3.1)
μ	σ unknown	$T_0 = (\bar{X} - \mu_0)/(S/\sqrt{n})$	(4.6)	(3.5)
σ^2	Normal pop.	$\chi_0^2 = (n-1)S^2/\sigma_0^2$	(4.7)	(3.6)
p	Large sample	$Z_0 = (\hat{p} - p_0)/\sqrt{p_0(1-p_0)/n}$	(4.8)	(3.7)
$\mu_1 - \mu_2$	σ_1, σ_2 known	Two-sample Z	(5.1)	(5.2)
$\mu_1 - \mu_2$	$\sigma_1^2 = \sigma_2^2$ unknown	Pooled t, S_p^2 from (5.3)	(5.4)	(5.5)
$\mu_1 - \mu_2$	$\sigma_1^2 \neq \sigma_2^2$ unknown	Welch's t, ν from (5.7)	(5.6)	—
μ_D	Paired data	Paired t: $T_0 = \bar{D}/(S_D/\sqrt{n})$	(5.8)	(5.9)
σ_1^2/σ_2^2	Normal pops.	$F_0 = S_1^2/S_2^2$	(5.13)	(5.14)
$p_1 - p_2$	Large samples	Two-prop Z (pooled $\hat{p}$)	(5.10)	(5.11)

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- **Normal probability plot:** points along a straight line $\Rightarrow$ normality is reasonable. Required for the χ^2 (4.7) and F (5.13) procedures.

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- **Small** n **+ skewed population:** neither z- nor t-interval is reliable. Collect more data.

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Common Z Critical Values (Quick Reference)

Table 3.1 in the textbook

Confidence Level	α	$\alpha/2$	$z_{\alpha/2}$
90%	0.10	0.05	1.645
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For one-sided tests/bounds: use z_α (not $z_{\alpha/2}$):

$$\alpha = 0.10 \Rightarrow z_\alpha = 1.282 \qquad \alpha = 0.05 \Rightarrow z_\alpha = 1.645$$
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For t, χ^2, and F: use the tables provided with the exam (Table B.3, Table B.4,

Good Luck on Major Exam 1!

Lecture 14: six fully worked practice problems