ISE 315: Engineering Statistics

Lecture 14 – Comprehensive Review: Solution Key

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CNC Milling Accuracy

Given: $n = 8$ paired observations (before/after recalibration).

Part (a): Compute differences

d_j = After − Before:

Machine	1	2	3	4	5	6	7	8	Sum
Before	85	78	92	88	76	95	82	90	
After	88	82	90	94	80	93	87	94	
d_j	3	4	-2	6	4	-2	5	4	22

$\bar{d} = \dfrac{22}{8} = \boxed{2.75}$

$\sum d_j^2 = 9 + 16 + 4 + 36 + 16 + 4 + 25 + 16 = 126$

$s_D^2 = \dfrac{\sum d_j^2 - n\bar{d}^2}{n-1} = \dfrac{126 - 8(2.75)^2}{7} = \dfrac{126 - 60.5}{7} = \dfrac{65.5}{7} = 9.357, \qquad s_D = \sqrt{9.357} = \boxed{3.059}$

Part (b): Hypothesis test (Paired t-test)

The same machines are measured before and after, so the observations are **dependent**. We use a paired t-test.

Step 1: $H_0\colon \mu_D = 0$ (no improvement)

Step 2: $H_1\colon \mu_D > 0$ (right-tailed; recalibration improved yield)

Step 3: $\alpha = 0.05$

Step 4: Test statistic: $T_0 = \dfrac{\bar{d}}{s_D/\sqrt{n}}$

Step 5: $T_0 = \dfrac{2.75}{3.059/\sqrt{8}} = \dfrac{2.75}{1.082} = \boxed{2.543}$

Step 6: Critical value: $t_{0.05,7} = 1.895$. Reject H_0 if $T_0 > 1.895$.

Step 7: $2.543 > 1.895 \Rightarrow$ **Reject** H_0.

There is sufficient evidence at the 5% significance level that recalibration significantly improved the mean dimensional accuracy (yield).

Part (c): P-value bounds

From the t-table with $\nu = 7$: $t_{0.025,7} = 2.365$ and $t_{0.01,7} = 2.998$.
Since $2.365 < T_0 = 2.543 < 2.998$:

$$\boxed{0.01 < P\text{-value} < 0.025} \qquad \text{(Exact: } P \approx 0.019)$$

Part (d): 95% CI for μ_D

$t_{0.025,7} = 2.365$

$$\bar{d} \pm t_{0.025,7} \cdot \frac{s_D}{\sqrt{n}} = 2.75 \pm 2.365(1.082) = 2.75 \pm 2.559$$

$$\boxed{95\%\text{ CI}: \ (0.191,\ 5.309)}$$

The CI does **not** contain zero $\Rightarrow$ consistent with rejecting H_0 in part (b).

AI Radiology Screening

Given: $n = 250$, $x = 40$, so $\hat{p} = 40/250 = 0.16$.

Part (a): 95% CI for p

Large-sample check: $n\hat{p} = 40 \geq 10$ ✓ and $n(1-\hat{p}) = 210 \geq 10$ ✓
$z_{0.025} = 1.96$

$$E = z_{0.025}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 1.96\sqrt{\frac{0.16(0.84)}{250}} = 1.96(0.02319) = 0.0454$$

$$\boxed{95\%\text{ CI}:\ 0.16 \pm 0.0454 = (0.1146,\ 0.2054)}$$

Part (b): Sample size with prior $\hat{p} = 0.16$

$$n = \left\lceil \frac{z_{0.025}^2\,\hat{p}(1-\hat{p})}{E^2} \right\rceil = \left\lceil \frac{(1.96)^2(0.16)(0.84)}{(0.04)^2} \right\rceil = \left\lceil \frac{0.5161}{0.0016} \right\rceil = \lceil 322.69 \rceil = \boxed{323}$$

Part (c): Sample size without prior

$$n = \left\lceil \frac{z_{0.025}^2 \times 0.25}{E^2} \right\rceil = \left\lceil \frac{(1.96)^2(0.25)}{(0.04)^2} \right\rceil = \left\lceil \frac{0.9604}{0.0016} \right\rceil = \lceil 600.25 \rceil = \boxed{601}$$

Part (d): Why is part (c) larger?

The conservative formula uses $\hat{p} = 0.5$, which maximizes $\hat{p}(1-\hat{p}) = 0.25$. Since the actual estimate gives $0.16 \times 0.84 = 0.1344 < 0.25$, the conservative approach overestimates the required variability and thus demands a larger sample to guarantee the margin of error for any possible p.

Thermal Management System

Given: $n = 36$, $\bar{x} = 104$, $\mu_0 = 100$, $\sigma^2 = 144$ ($\sigma = 12$, known).

Part (a): Hypothesis test (Z-test, σ known)

Step 1: $H_0\colon \mu = 100$

Step 2: $H_1\colon \mu > 100$ (right-tailed; thermal system increases runtime)

Step 3: $\alpha = 0.05$

Step 4: Test statistic: $Z_0 = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$

Step 5: $Z_0 = \dfrac{104 - 100}{12/\sqrt{36}} = \dfrac{4}{2} = \boxed{2.00}$

Step 6: Critical value: $z_{0.05} = 1.645$. Reject H_0 if $Z_0 > 1.645$.

Step 7: $2.00 > 1.645 \Rightarrow$ **Reject H_0**.

At the 5% significance level, there is sufficient evidence that the thermal management system has increased the mean battery runtime beyond 100 minutes.

Part (b): Practical interpretation

The observed mean runtime of 104 minutes is statistically significantly higher than the historical average of 100 minutes. The data supports that the new thermal management system is effective in extending battery life for industrial inspection drones.

Part (c): Type II error at $\mu = 106$

The critical value in terms of $\bar{X}$ (reject when $\bar{X} > \bar{x}_c$):

$$\bar{x}_c = \mu_0 + z_\alpha \cdot \frac{\sigma}{\sqrt{n}} = 100 + 1.645(2) = 103.29$$

We fail to reject H_0 when $\bar{X} \le 103.29$. If $\mu = 106$:

$$\begin{aligned}\beta &= P(\bar{X} \le 103.29 \mid \mu = 106) = P\left(Z \le \frac{103.29 - 106}{2}\right) \\ &= P(Z \le -1.355) = \Phi(-1.355) = \boxed{0.0877}\end{aligned}$$

The probability of failing to detect the improvement (when $\mu = 106$) is about 8.8%.

Part (d): Sample size for power = 0.90

$z_\alpha = z_{0.05} = 1.645$, $z_\beta = z_{0.10} = 1.282$ (since power $= 1 - \beta = 0.90 \Rightarrow \beta = 0.10$), $\delta = 106 - 100 = 6$.

$$n = \left\lceil \frac{(z_\alpha + z_\beta)^2 \sigma^2}{\delta^2} \right\rceil = \left\lceil \frac{(1.645 + 1.282)^2 (144)}{6^2} \right\rceil = \left\lceil \frac{(2.927)^2 (144)}{36} \right\rceil = \left\lceil \frac{1233.5}{36} \right\rceil = \lceil 34.27 \rceil = \boxed{35}$$

Industrial Circuit Board

Given: $n = 400$, $x = 52$, $p_0 = 0.10$, so $\hat{p} = 52/400 = 0.13$.

Part (a): Hypothesis test (Proportion Z-test)

Step 1: $H_0 : p = 0.10$

Step 2: $H_1 : p > 0.10$ (right-tailed; suspect higher defect rate)

Step 3: $\alpha = 0.05$

Step 4: Test statistic: $Z_0 = \dfrac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)/n}}$

Step 5: $Z_0 = \dfrac{0.13 - 0.10}{\sqrt{0.10(0.90)/400}} = \dfrac{0.03}{0.015} = \boxed{2.00}$

Step 6: Critical value: $z_{0.05} = 1.645$. Reject H_0 if $Z_0 > 1.645$.

Step 7: $2.00 > 1.645 \Rightarrow$ **Reject H_0**.

Sufficient evidence at 5% that the defect rate exceeds the claimed 10%.

Part (b): P-value

$$P = P(Z > 2.00) = 1 - \Phi(2.00) = 1 - 0.9772 = \boxed{0.0228}$$

Part (c): Would conclusion change at $\alpha = 0.01$?

At $\alpha = 0.01$: $z_{0.01} = 2.326$ and $Z_0 = 2.00 < 2.326$, so we **fail to reject** H_0. Equivalently, $P = 0.0228 > 0.01 = \alpha$. The conclusion **changes** — we no longer have sufficient evidence at the 1% level.

Part (d): 95% CI for p

$$E = 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 1.96\sqrt{\frac{0.13(0.87)}{400}} = 1.96(0.01682) = 0.0330$$

$$\boxed{95\%\text{ CI}: \; 0.13 \pm 0.0330 = (0.0970,\; 0.1630)}$$

Part (e): Is the CI consistent with the test?

The CI **contains** $p_0 = 0.10$, which may appear to conflict with rejecting H_0 in part (a).

Why the apparent inconsistency? The hypothesis test is **one-sided** and uses $p_0 = 0.10$ in the standard error, while the confidence interval is **two-sided** and uses $\hat{p} = 0.13$ in the standard error. These different standard errors can produce slight disagreement near the decision boundary. This is a well-known subtlety and does not indicate an error.

Tensile Strength of Polymer Specimens

Given: $n_1 = n_2 = 10$, $\bar{x}_1 = 85$, $\bar{x}_2 = 80$, $s_1 = 4$, $s_2 = 6$. Equal variances assumed.

Part (a): Hypothesis test (Pooled two-sample t-test)

Pooled variance:

$$s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2} = \frac{9(16)+9(36)}{18} = \frac{144+324}{18} = \frac{468}{18} = 26, \qquad s_p = \sqrt{26} = 5.099$$

Step 1: $H_0\colon \mu_1 - \mu_2 = 0$

Step 2: $H_1\colon \mu_1 - \mu_2 > 0$ (right-tailed; Process A has higher mean)

Step 3: $\alpha = 0.05$

Step 4: $T_0 = \dfrac{(\bar{x}_1 - \bar{x}_2) - 0}{s_p\sqrt{1/n_1 + 1/n_2}}$

Step 5: $T_0 = \dfrac{85-80}{5.099\sqrt{1/10+1/10}} = \dfrac{5.0}{5.099 \times 0.4472} = \dfrac{5.0}{2.280} = \boxed{2.193}$

Step 6: $\nu = n_1 + n_2 - 2 = 18$. Critical value: $t_{0.05,18} = 1.734$. Reject if $T_0 > 1.734$.

Step 7: $2.193 > 1.734 \Rightarrow$ **Reject** H_0.

Process A produces specimens with significantly higher mean tensile strength than Process B at the 5% significance level.

Part (b): P-value bounds

From the t-table with $\nu = 18$: $t_{0.025,18} = 2.101$ and $t_{0.01,18} = 2.552$.
Since $2.101 < T_0 = 2.193 < 2.552$:

$$\boxed{0.01 < P\text{-value} < 0.025} \qquad (\text{Exact: } P \approx 0.021)$$

Part (c): 95% CI for $\mu_1 - \mu_2$

$t_{0.025,18} = 2.101$

$$(\bar{x}_1 - \bar{x}_2) \pm t_{0.025,18} \cdot s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = 5.0 \pm 2.101(2.280) = 5.0 \pm 4.791$$

$$\boxed{95\%\ \text{CI}:\ (0.209,\ 9.791)}$$

The CI does **not** contain zero $\Rightarrow$ consistent with rejecting H_0.

Automated Vial Filling Machine

Given: $n = 20$, $s^2 = 8.0$ mL^2, $\nu = n - 1 = 19$.

Part (a): 95% CI for σ^2 and σ

From the χ^2-table: $\chi^2_{0.025,19} = 32.852$ and $\chi^2_{0.975,19} = 8.907$.

$$\text{CI for } \sigma^2\colon \left(\frac{(n-1)s^2}{\chi^2_{0.025,19}}, \frac{(n-1)s^2}{\chi^2_{0.975,19}}\right) = \left(\frac{152}{32.852}, \frac{152}{8.907}\right) = \boxed{(4.627,\ 17.066)}$$

$$\boxed{\text{CI for } \sigma\colon (\sqrt{4.627},\ \sqrt{17.066}) = (2.151,\ 4.131)}$$

Part (b): Test $H_0\colon \sigma^2 = 4$ vs $H_1\colon \sigma^2 \neq 4$

Since $\sigma_0^2 = 4$ is **not** inside the 95% CI $(4.627,\ 17.066)$, we **reject H_0** at $\alpha = 0.05$. The true variance is significantly different from 4.

Part (c): Test $H_0\colon \sigma^2 = 5$ vs $H_1\colon \sigma^2 > 5$ (right-tailed χ^2-test)

Step 1: $H_0\colon \sigma^2 = 5$

Step 2: $H_1\colon \sigma^2 > 5$ (right-tailed; variance exceeds specification)

Step 3: $\alpha = 0.05$

Step 4: Test statistic: $\chi_0^2 = \dfrac{(n-1)s^2}{\sigma_0^2}$

Step 5: $\chi_0^2 = \dfrac{19 \times 8.0}{5} = \dfrac{152}{5} = \boxed{30.4}$

Step 6: Critical value: $\chi^2_{0.05,19} = 30.144$. Reject if $\chi_0^2 > 30.144$.

Step 7: $30.4 > 30.144 \Rightarrow$ **Reject H_0**.

At the 5% level, there is evidence that the variance exceeds the 5 mL^2 regulatory specification. The machine does **not** meet the requirement.

Part (d): P-value and effect of $\alpha = 0.01$

From the χ^2-table with $\nu = 19$: $\chi^2_{0.05,19} = 30.144$ and $\chi^2_{0.025,19} = 32.852$.
Since $30.144 < \chi_0^2 = 30.4 < 32.852$:

$$\boxed{0.025 < P\text{-value} < 0.05} \qquad (\text{Exact: } P \approx 0.047)$$

At $\alpha = 0.01$: $\chi^2_{0.01,19} = 36.191$ and $30.4 < 36.191$, so we **fail to reject** H_0. The conclusion **changes** — at the more stringent 1% level, we cannot conclude the variance exceeds the specification.

Conceptual Problems – Solutions

Q1. An engineer constructs a 95% one-sided upper confidence bound for the mean wall thickness and obtains $\mu \leq 5.38$ mm. Which of the following is a correct interpretation?

(a) The true mean is definitely less than 5.38 mm.

(b) There is a 95% probability that $\mu = 5.38$ mm.

(c) We are 95% confident the true mean does not exceed 5.38 mm.

(d) 95% of all individual thicknesses are below 5.38 mm.

(e) The sample mean is 5.38 mm.

Answer: (c). A confidence bound is a statement about the *procedure's* long-run reliability: if we repeated this procedure many times, 95% of the resulting bounds would capture the true μ. Option (a) claims certainty, which is wrong. Option (b) assigns probability to a fixed parameter. Option (d) confuses a bound on μ with individual observations. Option (e) is unsupported — $\bar{x}$ is not necessarily 5.38.

Q2. An engineer constructs both a 90% and a 99% confidence interval for the mean fill volume from the same data with known σ. Compared to the 90% interval, how will the 99% interval differ?

(a) It will be narrower with the same center.

(b) It will be wider with a different center.

(c) It will be wider with the same center.

(d) It will have the same width but higher confidence.

(e) It will be narrower because 99% confidence requires more precision.

Answer: (c). Both intervals are centered at the same $\bar{x}$. The 99% interval uses $z_{0.005} = 2.576$ versus $z_{0.05} = 1.645$ for 90%, producing a wider margin of error. Higher confidence requires a wider interval to maintain coverage, not a narrower one.

Q3. A process engineer wants to construct a confidence interval for the variance σ^2 of a manufacturing process. The sample of $n = 15$ measurements appears to come from a non-normal (bimodal) distribution. Which statement is most appropriate?

(a) The χ^2-interval is valid because $n = 15$ is large enough for the CLT to apply.

(b) The χ^2-interval requires normality and should not be used here.

(c) The χ^2-interval is robust to non-normality and can still be used.

(d) A t-interval should be used for the variance when the population is non-normal.

(e) A z-interval for σ^2 can be used since σ is being estimated.

Answer: (b). The χ^2 distribution for $(n-1)S^2/\sigma^2$ is derived *exactly* from the assumption that the data come from a normal population. Unlike t- and z-procedures for the mean (which benefit from CLT robustness), the χ^2 inference for the variance is **not robust** to non-normality. A bimodal distribution is a clear violation.

Q4. A hypothesis test yields a P-value of 0.03. Which of the following is the correct interpretation?

(a) H_0 is definitely false.

(b) There is a 3% probability that H_0 is true.

(c) If H_0 is true, there is a 3% chance of observing data this extreme or more extreme.

(d) The probability of a Type II error is 0.97.

(e) The power of the test is 97%.

Answer: (c). The P-value is defined as $P(\text{data this extreme or more extreme} \mid H_0 \text{ is true})$. It is **not** the probability that H_0 is true (that would be a Bayesian posterior). Option (a) overstates the conclusion. Options (d) and (e) confuse P-value with β and power, which are unrelated quantities.

Q5. A researcher tests $H_0: p = 0.4$ versus $H_1: p \neq 0.4$ at $\alpha = 0.05$. The resulting 95% confidence interval for p is $[0.433,\ 0.466]$. What can you conclude about the P-value?

(a) P-value $= \alpha$.

(b) P-value $= 1 - \alpha$.

(c) P-value $< \alpha$.

(d) P-value $> \alpha$.

(e) More information is required to determine the relationship.

Answer: (c). The CI $[0.433, 0.466]$ does **not contain** $p_0 = 0.4$, so the two-sided test rejects H_0 at $\alpha = 0.05$. By the CI–test duality: rejecting at level α means P-value $< \alpha = 0.05$.

Q6. An engineer performs a one-sided Z-test with $H_0: \mu = 50$ versus $H_1: \mu > 50$ using σ known. The computed test statistic is $Z_0 = 1.50$ and $\alpha = 0.05$. What is the correct conclusion?

(a) Reject H_0 because $1.50 > 0.05$.

(b) Reject H_0 because $1.50 > 1.28$.

(c) Fail to reject H_0 because $1.50 < 1.645$.

(d) Fail to reject H_0 because $1.50 < 1.96$.

(e) Reject H_0 because $P < 0.05$.

Answer: (c). For a **right-tailed** test at $\alpha = 0.05$, the critical value is $z_{0.05} = 1.645$ (not $z_{0.025} = 1.96$, which is for a two-sided test). Since $Z_0 = 1.50 < 1.645$, we fail to reject. Option (a) incorrectly compares the test statistic to α itself. Option (b) uses $z_{0.10} = 1.28$, the wrong α. Option (e) is false: $P = P(Z > 1.50) = 0.0668 > 0.05$.

Q7. In a pooled t-test with sample sizes $n_1 = 12$ and $n_2 = 15$, what are the degrees of freedom for the test?

(a) $\min(11, 14) = 11$

(b) $n_1 + n_2 = 27$

(c) $n_1 + n_2 - 2 = 25$

(d) $n_1 - 1 = 11$

(e) $(n_1 - 1)(n_2 - 1) = 154$

Answer: (c). The pooled t-test uses both samples to estimate a common variance, so the degrees of freedom are $\nu = n_1 + n_2 - 2 = 12 + 15 - 2 = 25$. Option (a) is a conservative approximation sometimes used for Welch's test, not the pooled test. Option (b) forgets to subtract 2. Option (d) only accounts for one sample.

Q8. An engineer measures the fuel efficiency of 10 vehicles before and after a tune-up. Which statistical test is most appropriate for analyzing these data?

(a) Two-sample Z-test

(b) Pooled t-test

(c) Welch's t-test

(d) Paired t-test

(e) F-test

Answer: (d). The *same* 10 vehicles are measured before and after, making the observations **dependent** (paired). The correct approach is to analyze the differences $d_j = \text{After}_j - \text{Before}_j$ using a one-sample t-test on $\bar{d}$. The pooled and Welch t-tests assume independent samples and would be incorrect here.

Q9. Which of the following correctly describes the pooled variance s_p^2 in a two-sample t-test?

(a) It is the simple average of s_1^2 and s_2^2.

(b) It is a weighted average of s_1^2 and s_2^2, weighted by their degrees of freedom.

(c) It is always equal to s_1^2 when $n_1 = n_2$.

(d) It is the variance of the combined (merged) data set.

(e) It equals $s_1^2 + s_2^2$.

Answer: (b). The pooled variance is $s_p^2 = \frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}$, a weighted average with weights (n_1-1) and (n_2-1). When $n_1 = n_2$, it reduces to the simple average $(s_1^2+s_2^2)/2$, so (a) is only correct for equal sample sizes. Option (c) is wrong unless $s_1 = s_2$. Option (d) describes the merged variance, which differs because it ignores the different group means.

Q10. An engineer performs a two-sided F-test for equality of variances and obtains $F_0 = 2.10$ with $(\nu_1, \nu_2) = (9, 14)$. The critical value is $f_{0.025,9,14} = 3.21$. What is the correct conclusion?

(a) Reject H_0 because $F_0 > 1$.

(b) Fail to reject H_0; there is not enough evidence that the variances differ.

(c) Reject H_0 because $F_0 = 2.10$ exceeds the critical value.

(d) The test is invalid because $F_0 < f_{0.025}$.

(e) More information is needed to draw a conclusion.

Answer: (b). For a two-sided F-test at $\alpha = 0.05$, we reject if $F_0 > f_{0.025,\nu_1,\nu_2} = 3.21$. Since $F_0 = 2.10 < 3.21$, we do not reject. Option (a) is wrong — $F_0 > 1$ does not imply rejection; we must compare to the critical value. Option (c) incorrectly claims 2.10 exceeds 3.21.

Q11. A 95% confidence interval for $\mu_1 - \mu_2$ is $(-1.2,\ 3.8)$. If a two-sided hypothesis test of $H_0\colon \mu_1 = \mu_2$ is conducted at $\alpha = 0.05$, what is the correct decision?

(a) Reject H_0 because $3.8 > 0$.

(b) Fail to reject H_0 because the CI contains 0.

(c) Reject H_0 because the CI is asymmetric about 0.

(d) Reject H_0 because $|-1.2| < 3.8$.

(e) The decision cannot be determined without the test statistic t_0.

Answer: (b). By CI–test duality: if a two-sided $(1-\alpha)$ confidence interval **contains** the hypothesized value (here, $\mu_1 - \mu_2 = 0$), we **fail to reject** H_0 at level α. Since $-1.2 < 0 < 3.8$, zero is inside the interval. The asymmetry or endpoint magnitudes are irrelevant.

Q12. Two independent samples yield $n_1 = 100$, $x_1 = 30$ successes and $n_2 = 100$, $x_2 = 20$ successes. When testing $H_0\colon p_1 = p_2$, what is the correct pooled proportion $\hat{p}$?

(a) $\hat{p} = (0.30 + 0.20)/2 = 0.25$

(b) $\hat{p} = (30 + 20)/(100 + 100) = 0.25$

(c) $\hat{p} = \sqrt{0.30 \times 0.20} = 0.245$

(d) $\hat{p} = (0.30)(100/200) + (0.20)(100/200) = 0.25$

(e) Both (a) and (b) give the same answer, but (b) is the correct general formula.

Answer: (e). The correct general formula is $\hat{p} = (x_1 + x_2)/(n_1 + n_2)$, which pools the raw counts. When $n_1 = n_2$, averaging the sample proportions happens to give the same result, but when $n_1 \neq n_2$, only the count-based formula (b) is correct. For example, if $n_1 = 100, \hat{p}_1 = 0.30$ and $n_2 = 900, \hat{p}_2 = 0.20$, then $(0.30 + 0.20)/2 = 0.25$ but $(30 + 180)/1000 = 0.21$.

Q13. In a paired t-test with $n = 8$ pairs and $H_1 : \mu_D > 0$, the test statistic T_0 should be compared to which critical value?

(a) z_α

(b) $t_{\alpha,8}$

(c) $t_{\alpha,7}$

(d) $t_{\alpha/2,7}$

(e) $t_{\alpha,14}$

Answer: (c). A paired test reduces to a one-sample t-test on the $n = 8$ differences, so $\text{df} = n - 1 = 7$. Since $H_1 : \mu_D > 0$ is **one-sided**, we use $t_{\alpha,7}$ (not $t_{\alpha/2,7}$, which would be for a two-sided test). Option (a) uses the z-distribution, which requires known σ. Option (b) uses $\text{df} = 8$ instead of 7. Option (e) uses $\text{df} = 14$, as if the two samples were independent.

Q14. For a fixed sample size n, what happens to the probability of a Type II error (β) when the significance level α is increased?

(a) β increases.

(b) β decreases.

(c) β is unaffected by changes in α.

(d) β becomes equal to $1 - \alpha$.

(e) $\alpha + \beta = 1$ always.

Answer: (b). Increasing α widens the rejection region (we become more willing to reject H_0). This makes it easier to reject H_0 when it is false, so β (the probability of *failing* to reject a false H_0) **decreases**. This is the α–β tradeoff for fixed n. Note that $\alpha + \beta \neq 1$ in general — these are probabilities computed under different assumptions about the true parameter.

Q15. What is the effect on the power of a hypothesis test when the sample size n is increased while keeping α fixed?

(a) Power decreases because the test becomes too conservative.

(b) Power is unaffected because only α determines the rejection region.

(c) Power increases because β decreases with larger n.

(d) Power decreases because the standard error increases.

(e) Power remains unchanged because α is held constant.

Answer: (c). Increasing n decreases the standard error ($\mathrm{SE} = \sigma/\sqrt{n}$), which makes the sampling distribution of $\bar{X}$ tighter around the true mean. This allows the test to more easily distinguish the true parameter from H_0. Consequently, β decreases and power $= 1 - \beta$ increases. Option (d) is backwards — larger n *decreases* the standard error, not increases it.