ISE 315: Engineering Statistics

Lecture 2: Review of Estimation

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Based on Montgomery & Runger, Applied Statistics and Probability for Engineers, 6th Ed.

Lecture 2

Review of Estimation

Lecture 2 Outline: (Chapter 7.1-7.4)

- Point estimation (7.1)
- Sampling distributions (7.2)
- General concepts of point estimation (unbiased, standard error) (7.3) and Central Limit Theorem (7.2)
- (Next week) Methods of point estimation (method of moments, maximum likelihood) (7.4)

Chapter 7 Concept Review

Probability (ISE 205): Known $\rightarrow$ Observed

Population
(Parameters: μ, σ, p)

Sample
(Statistics: $\bar{x}, s, \hat{p}$)

Statistical Inference (ISE 315): Observed $\rightarrow$ Unknown

Given a random sample $X_1, X_2, \cdots, X_n$

Parameter Estimation
"What is μ?"

Hypothesis Testing
"Is $\mu = 50$?"

Point estimate: $\hat{\mu} = \bar{x}$
Interval estimate: $\bar{x} \pm t \cdot \frac{s}{\sqrt{n}}$

$H_0 : \mu = 50$ vs $H_1 : \mu \neq 50$
Decision: Reject or Fail to Reject

Key Terminologies

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3. **Point Estimator:**
   - A statistic $\hat{\theta}$ ("theta hat") used to estimate an unknown parameter θ ("theta")

$$\hat{\theta} = f(x_1, x_2, \ldots, x_n)$$

   - $\hat{\mu} = \bar{X}$ (sample average) estimates μ ("mu")
   - $\hat{\sigma}^2 = S^2$ (sample variance) estimates σ^2 ("sigma squared")

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- Probability distribution of a statistic
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Different samples yield different estimates—this variability is what we study!

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   - **Low/high Mean Squared Error (MSE):** Combines **both** variance & bias

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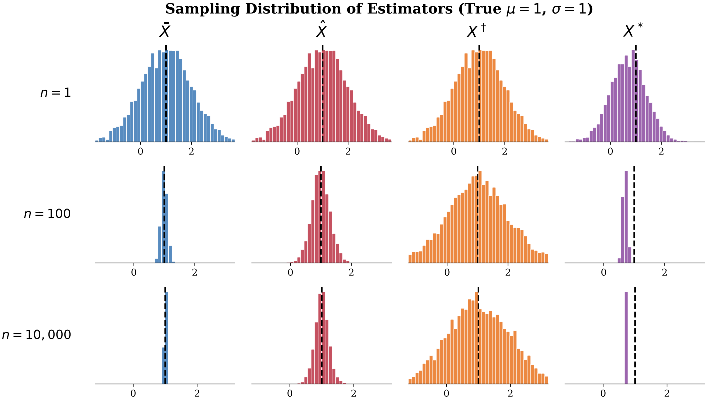
Ideal estimator: Low MSE (low variance and low bias)

Some estimators are better than others

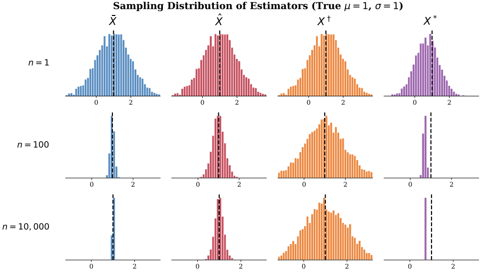
- Consider estimators $\bar{X}$, $\hat{X}$, $X^\dagger$, X^* for population mean μ:



Which one would you choose and why?

Some statistics are better than others

- What you have seen is the sampling distribution of
  - Sample mean: $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$
  - Half-range: $\hat{X} = \frac{\max(X_i)+\min(X_i)}{2}$
  - First sample: $X^{\dagger} = X_1$
  - Shrinkage: $X^{*} = 0.7\bar{X}$



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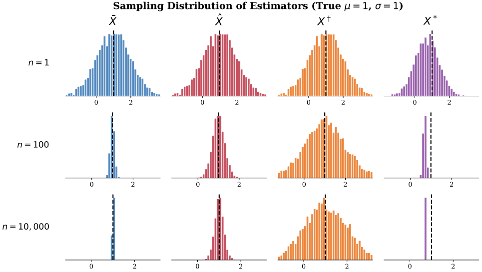

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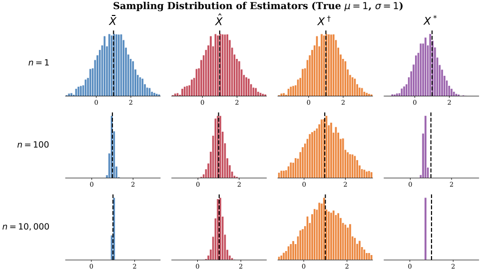

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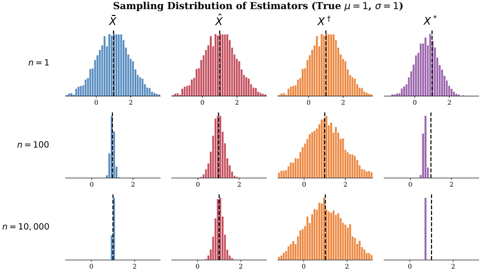
- Which one(s) has smallest variance?
- Which one has lowest MSE for $n = 1$? For larger n?



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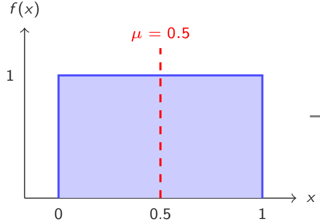
  - By CLT, $\bar{X}$ is **loosely approximately** $N(\mu_{\bar{X}}, \sigma_{\bar{X}}) = N(0.5, 0.091)$
  - Use Normal distribution properties for analysis (e.g., finding $P(\bar{X} > 0.6)$)

Population vs Sampling Distribution

Population Distribution

$X_i \sim \text{Uniform}(0, 1)$



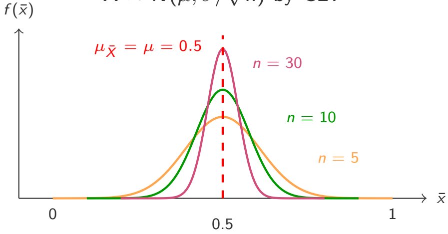
$\mu = 0.5, \quad \sigma^2 = 1/12$

Fixed shape (doesn't change)

Take many samples of size n

Sampling Distribution of $\bar{X}$

$\bar{X} \approx N(\mu, \sigma/\sqrt{n})$ by CLT



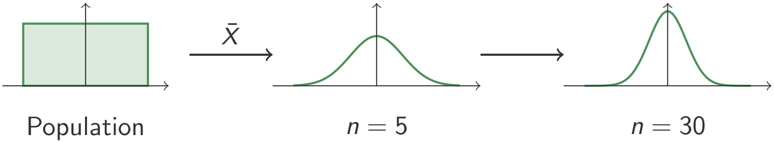
$\mu_{\bar{X}} = \mu, \quad \sigma^2_{\bar{X}} = \sigma^2/n$

Shape depends on n (gets narrower as $n \uparrow$)

Both have different distributions: **same mean**, but **different shape** and **variance**

Central Limit Theorem (CLT)

Visual illustration



As $n \uparrow$, distribution of $\bar{X}$ becomes Normal and more concentrated around μ.

Central Limit Theorem (CLT)

If $X_1, X_2, \ldots, X_n$ is a random sample of size n taken from a population with mean μ and variance σ^2, and if $\bar{X}$ is the sample mean

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i,$$

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Key insight: Regardless of the population distribution, $\bar{X}$ is approximately Normal for large n!

CLT for Z-statistic

Using the properties of the normal distribution, we can standardize $\bar{X}$ and calculate the Z-statistic:

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}.$$

For large n, the Z-statistic has a sampling distribution of the Standard Normal $N(0, 1)$.

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Key insight: Regardless of the population distribution, Z is approximately standard Normal for large n!

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- The probability distribution of a statistic is called the **sampling distribution**
- The **Central Limit Theorem (CLT)** is a theory that the sampling distribution of $\bar{X} \approx$ Normal for large n

Questions you might be asked about sampling distribution

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How to answer these questions?

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  - Sometimes, it is useful to find the standardized statistic $Z = \frac{\bar{X}-\mu}{\sigma_{\bar{X}}}$
  - Use table/calculator to find the probability using standard Normal $N(0, 1)$

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  - Sample proportion $\left(\hat{p} = \frac{X_1 + X_2 + \cdots + X_n}{n}\right)$
    (also mean but the random variables are binaries (0 or 1))

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  - Shifted mean $(\bar{X} - c)$
  - Scaled shifted mean $(a(\bar{X} - c))$—Z-statistic is an example of this
  - Sample proportion $\left(\hat{p} = \frac{X_1+X_2+\cdots+X_n}{n}\right)$
    (also mean but the random variables are binaries (0 or 1))
  - Difference in means $(\bar{X}_1 - \bar{X}_2)$ or proportions $(\hat{p}_1 - \hat{p}_2)$

Questions you might be asked about sampling distribution

What variants we might see?

- The population mean and variance might not be provided
- There are multiple populations and random samples involved
- The statistic is not the sample mean $\bar{X}$ but something else
  - Shifted mean $(\bar{X} - c)$
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Same approach:
population μ and σ^2 $\Rightarrow$ sampling dist $\mu_{\hat{\theta}}, \sigma^2_{\hat{\theta}}$ $\Rightarrow$ apply $N(\mu_{\hat{\theta}}, \sigma^2_{\hat{\theta}}) \Rightarrow Z \sim N(0,1)$

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- Mean μ (population average)

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Can you give examples why we care about these parameters?

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What estimators would you use for each parameter?

What parameters we care about in this class?

- Mean μ (population average) (sample mean $\bar{X}$)
- Variance σ^2 (spread of the data) (sample variance S^2)
- Proportion p (fraction of success in a population) (sample proportion $\hat{p}$)
- Difference in means $\mu_1 - \mu_2$ (effect size) $(\bar{X}_1 - \bar{X}_2)$
- Difference in proportions $p_1 - p_2$ (difference in success rates) $(\hat{p}_1 - \hat{p}_2)$

Summary #2

- Summary 1
- Summary 2

Class Announcement Week 1

- Syllabues updated, check Blackboard for the latest
- Homework 1 is due next Tuesday before class (in Gradescope)
- Office hour: Monday 9-10 AM (22-219 or Zoom, link in Blackboard)

Have a great first week!