ISE 315: Engineering Statistics

Supplementary Material: Finding P-Value Bounds from Statistical Tables

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Supplementary Material

Finding P-Value Bounds from the t, χ^2, and F Tables

Why Do We Need P-Value Bounds?

Software gives **exact p-values** (e.g., $p = 0.0297$), but statistical tables only give **critical values** at specific α levels.

From tables, we can only **bound** the p-value between two α levels.

On exams and quizzes, you are expected to find p-value bounds using:

- The **t-table** (Appendix Table V)
- The **χ^2-table** (Appendix Table IV)
- The **F-table** (Appendix Table VI)

Key idea: Locate your test statistic between two critical values, then read off the corresponding α values as your bounds.

The General Strategy: Three Steps

Step 1: Identify the correct table row/section

- For t: Go to the row matching your degrees of freedom
- For χ^2: Go to the row matching your degrees of freedom
- For F: Go to the page matching your α, then find (df_1, df_2)

Step 2: Bracket your test statistic between two critical values

- Find $t_{\alpha_1} < |t_0| < t_{\alpha_2}$ (or similarly for χ^2, F)

Step 3: Convert to a p-value bound

- **One-tailed:** $\alpha_2 < p\text{-value} < \alpha_1$
- **Two-tailed:** $2\alpha_2 < p\text{-value} < 2\alpha_1$

Part 1: P-Value Bounds from the t-Table

How the t-table is organized:

- Rows = degrees of freedom ($\nu = n - 1$)
- Columns = upper-tail area α: 0.25, 0.10, 0.05, 0.025, 0.01, 0.005
- Entry = $t_{\alpha,\nu}$ such that $P(T > t_{\alpha,\nu}) = \alpha$

Example: Row for $\nu = 14$:

$\nu \backslash \alpha$	0.25	0.10	0.05	0.025	0.01	0.005
14	0.692	1.345	1.761	2.145	2.624	2.977

Reading: $P(T_{14} > 2.145) = 0.025$ and $P(T_{14} > 2.624) = 0.01$.

t-Table Example 1

A petroleum engineer measures the hardness (HRC) of $n = 15$ drill bits from a new supplier. The sample gives $\bar{x} = 52.0$ and $s = 3.2$. Test whether the mean hardness differs from the specification of $\mu_0 = 50.0$ at $\alpha = 0.05$. Estimate the p-value.

Hypotheses: $H_0 : \mu = 50$ vs $H_1 : \mu \neq 50$. Two-tailed t-test, $df = 14$.

Test statistic: $t_0 = \dfrac{52.0 - 50.0}{3.2/\sqrt{15}} = \dfrac{2.0}{0.826} = 2.42$

P-value bounds: Row $\nu = 14$: $t_{0.025,14} = 2.145 < 2.42 < 2.624 = t_{0.01,14}$.
Upper tail: $0.01 < P(T > 2.42) < 0.025$. Two-tailed: $\boxed{0.02 < p\text{-value} < 0.05}$

Decision: Since $p < 0.05 = \alpha$, we **reject** H_0. The mean hardness differs from spec.

t-Table Example 2

A chemical engineer claims a new catalyst increases mean yield above 90 units. A sample of $n = 20$ batches gives $\bar{x} = 91.5$ and $s = 3.5$. Test at $\alpha = 0.05$. Estimate the p-value.

Hypotheses: $H_0 : \mu = 90$ vs $H_1 : \mu > 90$. One-tailed upper t-test, $df = 19$.

Test statistic: $t_0 = \dfrac{91.5 - 90.0}{3.5/\sqrt{20}} = \dfrac{1.5}{0.783} = 1.92$

P-value bounds: Row $\nu = 19$: $t_{0.05,19} = 1.729 < 1.92 < 2.093 = t_{0.025,19}$.
One-tailed: $\boxed{0.025 < p\text{-value} < 0.05}$

Decision: At $\alpha = 0.05$: $p < 0.05$, so **reject** H_0. Evidence supports the claim.
At $\alpha = 0.01$: $p > 0.01$, so **fail to reject**. Not significant at 1%.

Key Difference: One-Tailed vs. Two-Tailed from the t-Table

One-Tailed Test	**Two-Tailed Test**
Read α values directly from the column headers bracketing $\lvert t_0 \rvert$	Double the column headers to get the p-value bounds
If $t_{0.05} < \lvert t_0 \rvert < t_{0.025}$:	If $t_{0.025} < \lvert t_0 \rvert < t_{0.01}$:
$\Rightarrow 0.025 < p < 0.05$	$\Rightarrow 0.02 < p < 0.05$

This is the #1 mistake on exams: forgetting to multiply by 2 for two-tailed tests, or multiplying when the test is one-tailed.

Part 2: P-Value Bounds from the χ^2-Table

How the χ^2-table is organized:

- Rows $=$ degrees of freedom ($\nu = n - 1$)
- Columns $=$ **upper-tail area** α
- The table includes **both small and large** α values:

$$\underbrace{0.995,\ 0.99,\ 0.975,\ 0.95,\ 0.90}_{\text{These give \textbf{left-tail} critical values}} \quad \underbrace{0.10,\ 0.05,\ 0.025,\ 0.01,\ 0.005}_{\text{These give \textbf{right-tail} critical values}}$$

Important difference from the t-table:

- The χ^2 distribution is **not symmetric**
- For **lower-tail** tests, use columns with large α values $(0.95, 0.975, \ldots)$
- For **upper-tail** tests, use columns with small α values $(0.05, 0.025, \ldots)$

Reading the χ^2-Table: Upper vs. Lower Tail

Example row: $\nu = 19$

$\nu \backslash \alpha$	Left-tail critical values					Right-tail critical values				
	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582

Key readings:

- $\chi^2_{0.05,19} = 30.14$ means $P(\chi^2_{19} > 30.14) = 0.05$ (right tail)
- $\chi^2_{0.95,19} = 10.12$ means $P(\chi^2_{19} > 10.12) = 0.95$, so $P(\chi^2_{19} < 10.12) = 0.05$ (left tail)

χ^2-Table Example 1

A bottling plant specifies $\sigma^2 = 0.020$ mL^2 for fill volume. A sample of $n = 20$ bottles gives $s^2 = 0.032$ mL^2. Test whether the variance differs from the standard at $\alpha = 0.05$. Estimate the p-value.

Hypotheses: $H_0 : \sigma^2 = 0.020$ vs $H_1 : \sigma^2 \neq 0.020$. Two-tailed χ^2-test, $df = 19$.

Test statistic: $\chi_0^2 = \dfrac{(n-1)s^2}{\sigma_0^2} = \dfrac{19 \times 0.032}{0.020} = 30.4$ (falls in upper tail)

P-value bounds: Row $\nu = 19$: $\chi^2_{0.05,19} = 30.14 < 30.4 < 32.85 = \chi^2_{0.025,19}$.
Upper tail: $0.025 < P(\chi^2 > 30.4) < 0.05$. Two-tailed: $\boxed{0.05 < p\text{-value} < 0.10}$

Decision: Since $p > 0.05 = \alpha$, we **fail to reject** H_0. Not enough evidence.

χ^2-Table Example 2

A manufacturer claims a new CNC machine achieves lower variance than the current standard of $\sigma_0^2 = 0.50$ mm^2. A sample of $n = 25$ parts gives $s^2 = 0.30$ mm^2. Test at $\alpha = 0.05$. Estimate the p-value.

Hypotheses: $H_0 : \sigma^2 = 0.50$ vs $H_1 : \sigma^2 < 0.50$. One-tailed lower χ^2-test, $df = 24$.

Test statistic: $\chi_0^2 = \dfrac{24 \times 0.30}{0.50} = 14.4 \quad \text{(falls in lower tail)}$

P-value bounds: Row $\nu = 24$, use **large-α** columns:
$\chi^2_{0.95,24} = 13.85 < 14.4 < 15.66 = \chi^2_{0.90,24}$.
$P(\chi^2 < 14.4)$: between $1 - 0.95 = 0.05$ and $1 - 0.90 = 0.10$.
$\boxed{0.05 < p\text{-value} < 0.10}$

Decision: Since $p > 0.05 = \alpha$, we **fail to reject** H_0. Insufficient evidence.

Key Trick: Lower-Tail χ^2 Bounds

For a lower-tail χ^2 test:

If $\chi^2_{\alpha_1,\nu} < \chi^2_0 < \chi^2_{\alpha_2,\nu}$ where $\alpha_1 > \alpha_2$ (both large values like $0.90, 0.95$), then:

$$P(\chi^2 < \chi^2_0) \text{ is between } (1 - \alpha_1) \text{ and } (1 - \alpha_2)$$

$\alpha_1 = 0.95$, $\alpha_2 = 0.90 \;\Rightarrow\;$ p-value between 0.05 and 0.10

Think of it this way:

- The column header α always means $P(\chi^2 > \text{value}) = \alpha$
- So $P(\chi^2 < \text{value}) = 1 - \alpha$
- You just subtract from 1

Part 3: P-Value Bounds from the F-Table

How the F-table is organized (different from t and χ^2!):

- **Separate page for each α:** $\alpha = 0.25,\ 0.10,\ 0.05,\ 0.025,\ 0.01$
- On each page: rows $= df_2$ (denominator), columns $= df_1$ (numerator)
- Entry $= F_{\alpha,df_1,df_2}$ such that $P(F > F_{\alpha,df_1,df_2}) = \alpha$

To find p-value bounds:

- Check your test statistic across **multiple pages** (one per α)
- For each α page, look up F_{α,df_1,df_2}
- Find which two consecutive α values bracket your f_0

Tip: You are reading **across pages** instead of across columns.

F-Table Example 1

An engineer suspects Line A is more variable than Line B. Samples: $n_1 = 10$ from Line A with $s_1^2 = 0.057$, and $n_2 = 15$ from Line B with $s_2^2 = 0.020$. Test at $\alpha = 0.05$. Estimate the p-value.

Hypotheses: $H_0 : \sigma_1^2 = \sigma_2^2$ vs $H_1 : \sigma_1^2 > \sigma_2^2$. One-tailed F-test.
$df_1 = 9$, $df_2 = 14$.

Test statistic: $F_0 = \frac{s_1^2}{s_2^2} = \frac{0.057}{0.020} = 2.85$

F-Table Example 1

P-value bounds: Look up $F_{\alpha,9,14}$ across pages:

α	0.25	0.10	**0.05**	**0.025**	0.01
$F_{\alpha,9,14}$	1.47	2.12	**2.65**	**3.21**	4.03

Bracket: $F_{0.05} = 2.65 < 2.85 < 3.21 = F_{0.025}$. $\boxed{0.025 < p\text{-value} < 0.05}$

Decision: $p < 0.05$, so **reject** H_0. Line A has significantly greater variance.

F-Table Example 2

A process engineer tests whether two catalyst batches have **different** variances. Samples: $n_1 = 8$ with $s_1 = 4.8$, and $n_2 = 10$ with $s_2 = 2.7$. Test at $\alpha = 0.05$. Estimate the p-value.

Hypotheses: $H_0 : \sigma_1^2 = \sigma_2^2$ vs $H_1 : \sigma_1^2 \neq \sigma_2^2$. Two-tailed F-test.
$df_1 = 7$, $df_2 = 9$.

Test statistic: $F_0 = \dfrac{s_1^2}{s_2^2} = \dfrac{4.8^2}{2.7^2} = \dfrac{23.04}{7.29} = 3.16$

F-Table Example 2

P-value bounds: Look up $F_{\alpha,7,9}$ across pages:

α	0.25	**0.10**	**0.05**	0.025	0.01
$F_{\alpha,7,9}$	1.60	**2.51**	**3.29**	4.20	5.61

Upper tail: $0.05 < P(F > 3.16) < 0.10$. Two-tailed: $\boxed{0.10 < p\text{-value} < 0.20}$

Decision: $p > 0.05$, so **fail to reject** H_0. No significant difference in variances.

Quick Reference: P-Value Bounds Cheat Sheet

Test	One-Tailed Bound	Two-Tailed Bound	Table Tip
t-**test**	Read α values directly from columns bracketing $\|t_0\|$	Multiply both bounds by 2	Read across columns in one row
χ^2-**test** (upper)	Read α from columns bracketing χ_0^2	Multiply both bounds by 2	Use small-α columns
χ^2-**test** (lower)	Subtract column α from 1	Multiply by 2 after subtracting	Use large-α columns
F-**test**	Read α values from pages bracketing f_0	Multiply both bounds by 2	Read across **pages**

Common Mistakes to Avoid

- **Forgetting to multiply by 2 for two-tailed tests.**
  The table gives you the *one-tail* area only. Two-tailed p-value $= 2\times$ one-tail area.

- **Wrong direction for χ^2 lower-tail tests.**
  Column header α means $P(\chi^2 > \text{value})$. For lower tail: $p = 1 - \alpha$.

- **Reading the wrong F-table page.**
  Each page corresponds to a different α. Also: df_1 is columns, df_2 is rows.

- **Reversing the inequality direction.**
  If $t_{0.025} < |t_0| < t_{0.01}$, then $0.01 <$ tail area < 0.025.
  The *larger* critical value corresponds to the *smaller* α.

Summary

- The t-**table** and χ^2-**table** are read across columns for a given df row.
- The F-**table** is read across pages (each page $=$ different α).
- For **one-tailed** tests, the bounds come directly from the table.
- For **two-tailed** tests, multiply the one-tail bounds by 2.
- For χ^2 **lower-tail** tests, subtract the column header from 1.
- Always state your bounds as: $\alpha_{\text{small}} < p\text{-value} < \alpha_{\text{large}}$.

Practice with the textbook appendix tables until this becomes second nature. These bounds are helpful on every exam and quiz!