ISE 315: Engineering Statistics

Homework 2 Solution

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Problem 1 [15 pt] — Sampling Distribution of the Mean (CLT)

Given: $\mu = 12$ s, $\sigma = 3$ s, $n = 49$. Find $P(11 \le \bar{X} \le 13)$.

Solution:

Step 1: By CLT, $\bar{X} \sim N\big(\mu,\ \sigma^2/n\big) = N(12,\ 9/49)$.

Step 2: Standard error: $\text{SE} = 3/\sqrt{49} = 3/7 \approx 0.4286$.

Step 3: Standardize:

$$Z_1 = \frac{11 - 12}{3/7} = -\frac{7}{3} \approx -2.3333, \qquad Z_2 = \frac{13 - 12}{3/7} = \frac{7}{3} \approx 2.3333.$$

Step 4: Look up:

$$P(11 \le \bar{X} \le 13) = \Phi(2.33) - \Phi(-2.33) = 0.9901 - 0.0099 = 0.9803.$$

$$\boxed{P(11 \le \bar{X} \le 13) \approx 0.9804}$$

Note: Accept any answer in the range 0.98–0.9806 due to rounding.

Scoring Rubric

Error	**Deduction**
Does not use $\sigma/\sqrt{n}$; uses σ directly	-6
Arithmetic error in SE (e.g., 3/49 instead of 3/7)	-3
Standardization error (wrong Z-values)	-4
Correct Z-values but wrong table lookup or subtraction	-3
Does not invoke CLT or state distribution of $\bar{X}$	-2
Missing or no work shown	-12

Problem 2 [15 pt] — Sampling Distribution (Drone Range)

Given: $X \sim N(\mu = 40, \sigma = 14)$, $n = 49$. Find $P(38 \le \bar{X} \le 42)$.

Solution:

Step 1: $\bar{X} \sim N(40,\ 196/49) = N(40,\ 4)$, so SE $= 14/\sqrt{49} = 2$.

Step 2: Standardize:

$$Z_1 = \frac{38 - 40}{2} = -1, \qquad Z_2 = \frac{42 - 40}{2} = 1.$$

Step 3: Look up:

$$P(38 \le \bar{X} \le 42) = \Phi(1) - \Phi(-1) = 0.8413 - 0.1587 = 0.6827.$$

$$\boxed{P(38 \le \bar{X} \le 42) \approx 0.6827}$$

Scoring Rubric

Error	Deduction
Does not use $\sigma/\sqrt{n}$; uses $\sigma = 14$ directly	-6
Arithmetic error in SE	-3
Wrong Z-values from correct SE	-3
Correct Z-values but wrong table lookup or subtraction	-2
Missing or no work shown	-12

Problem 3 [10 pt] — Sample Size for Standard Error

Given: $X \sim N(\mu = 50, \sigma^2 = 64)$, want $\text{SE}(\bar{X}) \le 2$ ms.

Solution:

Step 1: Identify $\sigma = \sqrt{64} = 8$ ms.

Step 2: The standard error of the sample mean is $\text{SE}(\bar{X}) = \dfrac{\sigma}{\sqrt{n}}$.

Step 3: Set up the inequality:

$$\frac{8}{\sqrt{n}} \le 2 \quad \Longrightarrow \quad \sqrt{n} \ge \frac{8}{2} = 4 \quad \Longrightarrow \quad n \ge 16.$$

$$\boxed{n \ge 16}$$

Scoring Rubric

Error	Deduction
Uses $\sigma^2 = 64$ instead of $\sigma = 8$ (e.g., gets $n \ge 1024$)	-4
Incorrect SE formula (e.g., σ/n instead of $\sigma/\sqrt{n}$)	-4
Algebra error in solving the inequality	-3
Correct setup but does not state final answer for n	-2
Does not round up to an integer (if non-integer arose)	-1
Missing or no work shown	-8

Problem 4 [15 pt] — Difference of Two Sample Means

Given: AGVs: $n_1 = 25$, $\mu_1 = 120$, $\sigma_1 = 10$. Conveyors: $n_2 = 16$, $\mu_2 = 110$, $\sigma_2 = 12$. Find $P(5 \le \bar{X}_1 - \bar{X}_2 \le 15)$.

Solution:

Step 1: Mean of the difference:

$$E[\bar{X}_1 - \bar{X}_2] = \mu_1 - \mu_2 = 120 - 110 = 10.$$

Step 2: Variance of the difference (independent samples):

$$\mathrm{Var}(\bar{X}_1 - \bar{X}_2) = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2} = \frac{100}{25} + \frac{144}{16} = 4 + 9 = 13.$$

Step 3: Standard error: $\mathrm{SE} = \sqrt{13} \approx 3.6056$.

Step 4: Standardize:

$$Z_1 = \frac{5 - 10}{\sqrt{13}} = \frac{-5}{\sqrt{13}} \approx -1.3868, \qquad Z_2 = \frac{15 - 10}{\sqrt{13}} = \frac{5}{\sqrt{13}} \approx 1.3868.$$

Step 5: Look up:

$$P(5 \le \bar{X}_1 - \bar{X}_2 \le 15) = \Phi(1.39) - \Phi(-1.39) \approx 0.9177 - 0.0823 = 0.8345.$$

$$\boxed{P(5 \le \bar{X}_1 - \bar{X}_2 \le 15) \approx 0.8345}$$

Note: Accept any answer in the range 0.83–0.84 due to rounding in table lookups.

Scoring Rubric

Error	Deduction
Wrong mean of difference (e.g., forgets subtraction)	-3
Incorrect variance formula (e.g., subtracts variances)	-5
Uses σ instead of $\sigma/\sqrt{n}$ for each term	-5
Correct SE but wrong standardization	-3
Correct Z-values but wrong table lookup or subtraction	-2
Missing or no work shown	-12

Problem 5 [10 pt] — Deviation Probability (T-distribution)

Given: $X \sim N(\mu, \sigma^2)$ unknown μ, $n = 16$, $\bar{x} = 28$ m, $S^2 = 25$ m^2. Find $P(|\bar{X} - \mu| > 5)$.

Solution:

Step 1: Since σ is unknown and we use S, the pivot is

$$T = \frac{\bar{X} - \mu}{S/\sqrt{n}} \sim t_{n-1} = t_{15}.$$

Step 2: Compute $S = \sqrt{25} = 5$ and $S/\sqrt{n} = 5/\sqrt{16} = 1.25$.

Step 3: Standardize:

$$P(|\bar{X} - \mu| > 5) = P\left(|T| > \frac{5}{1.25}\right) = P(|T| > 4.0).$$

Step 4: From the t-table with 15 df, $t_{0.0005,15} \approx 3.733$ and $t_{0.001,15} \approx 4.073$. Since 4.0 falls between these, the two-tailed probability is approximately 0.001.

$$\boxed{P(|\bar{X} - \mu| > 5) \approx 0.0012 \approx 0}$$

Note: Accept "≈ 0" or any answer near 0.001.

Scoring Rubric

Error	**Deduction**
Uses Z-distribution instead of t-distribution	-4
Uses $\sigma = 25$ instead of $S = \sqrt{25} = 5$	-3
Incorrect standardization (wrong denominator)	-3
Correct t-value but misreads table / wrong conclusion	-2
Missing or no work shown	-8

Problem 6 [15 pt] — Difference of Proportions

Given: $n_1 = n_2 = 100$, $X_1 = 90$, $X_2 = 75$, $p_1 = 0.90$, $p_2 = 0.75$.

Part (a) [5 pt]: Point Estimate

The unbiased point estimate for $p_1 - p_2$ is:

$$\hat{p}_1 - \hat{p}_2 = \frac{X_1}{n_1} - \frac{X_2}{n_2} = \frac{90}{100} - \frac{75}{100} = 0.90 - 0.75 = 0.15.$$

$$\boxed{\hat{p}_1 - \hat{p}_2 = 0.15}$$

Rubric — Part (a)

Error	Deduction
Wrong formula or wrong arithmetic	-3
No work shown (answer only)	-2

Part (b) [5 pt]: Standard Error

Using the **true** proportions (given in the problem):

$$\begin{aligned} \text{SE}(\hat{p}_1 - \hat{p}_2) &= \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}} \\ &= \sqrt{\frac{0.90 \times 0.10}{100} + \frac{0.75 \times 0.25}{100}} \\ &= \sqrt{0.0009 + 0.001875} \\ &= \sqrt{0.002775} \\ &\approx 0.0527. \end{aligned}$$

$$\boxed{\text{SE} \approx 0.0527}$$

Rubric — Part (b)

Error	Deduction
Wrong SE formula (e.g., missing one term, wrong structure)	-3
Uses $\hat{p}$ instead of p (acceptable if noted, but less precise)	-1
Arithmetic error in computation	-2

Part (c) [5 pt]: Probability of Large Deviation

Find $P(|(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)| > 0.10)$.

By normal approximation, $\hat{p}_1 - \hat{p}_2 \mathrel{\dot\sim} N(p_1 - p_2,\ \mathrm{SE}^2)$, so:

$$Z = \frac{0.10}{0.0527} \approx 1.898.$$

$$P(|Z| > 1.898) = 2\big[1 - \Phi(1.898)\big] = 2(1 - 0.9712) = 2(0.0288) \approx 0.0577.$$

$$\boxed{P \approx 0.0577}$$

Note: Accept any answer in the range 0.055–0.060 due to rounding.

Rubric — Part (c)

Error	**Deduction**
Wrong setup (e.g., one-sided instead of two-sided)	-3
Uses wrong SE from part (b) but correct method	-1
Arithmetic or table-lookup error	-2

Total: 15 + 15 + 10 + 15 + 10 + 15 = **80 points.**