ISE 315: Engineering Statistics

Homework 3 Solution

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Set A

Problem 1 [12 pt] — T-Interval for the Mean (Unknown σ)

Given: $n = 15$, $\bar{x} = 125$ N, $s = 18$ N, 95% CI.

Solution:

Step 1: Degrees of freedom: $\nu = 14$. For 95% CI, $t_{0.025,14} = 2.145$.

Step 2: Standard error: $\text{SE} = 18/\sqrt{15} = 4.6476$.

Step 3: Margin of error: $2.145 \times 4.6476 = 9.97$.

$$125 \pm 9.97 \quad \Longrightarrow \quad \boxed{(115.03,\ 134.97)}$$

Scoring Rubric

Error	Deduction
Uses z instead of t (unknown σ)	-4
Wrong degrees of freedom	-2
Wrong t-value from table	-2
Arithmetic error	-2
Missing or no work shown	-10

Problem 2 [15 pt] — Proportion CI and Compliance Assessment

Given: $n = 250$, $x = 237$, 95% CI.

Solution:

Step 1: Point estimate: $\hat{p} = 237/250 = 0.948$.

Step 2: Standard error: $\text{SE} = \sqrt{0.948 \times 0.052/250} = 0.0140$.

(a) 95% CI ($z_{0.025} = 1.96$):

$$0.948 \pm 1.96(0.0140) = 0.948 \pm 0.0275 \quad \Longrightarrow \quad \boxed{(0.9205,\ 0.9755)}$$

(b) The lower bound is $0.9205 > 0.92$, so the entire interval lies above 92%. The manufacturer **can claim compliance** at the 95% confidence level.

Note: The margin is narrow (0.9205 vs. 0.92). Accept either "barely supports" or "supports."

Scoring Rubric

Error	Deduction
Wrong $\hat{p}$ or SE computation	-3
Wrong z-value for 95%	-2
Part (b): wrong conclusion or no justification	-4
Arithmetic error	-2
Missing or no work shown	-12

Problem 3 [12 pt] — Z-Interval for the Mean (Known σ)

Given: $\sigma = 4.2$ ppm, $n = 30$, $\bar{x} = 18.5$ ppm, 90% CI.

Solution:

Step 1: Standard error: $\text{SE} = 4.2/\sqrt{30} = 0.7668$.

Step 2: 90% CI ($z_{0.05} = 1.645$):

$$18.5 \pm 1.645(0.7668) = 18.5 \pm 1.261 \quad \Longrightarrow \quad \boxed{(17.24,\ 19.76)}$$

Scoring Rubric

Error	Deduction
Uses t instead of z (σ is known)	-4
Wrong z-value for 90%	-2
Wrong SE computation	-3
Arithmetic error	-2
Missing or no work shown	-10

Problem 4 [15 pt] — Chi-Square Interval for Variance

Given: $n = 22$, $s^2 = 0.64$, $\nu = 21$, 99% CI.

Solution:

Step 1: From the χ^2 table with $\nu = 21$: $\chi^2_{0.005,21} = 38.932$ and $\chi^2_{0.995,21} = 8.034$.

(a) 99% CI for σ^2:

$$\left(\frac{21 \times 0.64}{38.932},\ \frac{21 \times 0.64}{8.034}\right) = \left(\frac{13.44}{38.932},\ \frac{13.44}{8.034}\right) = \boxed{(0.3452,\ 1.6729)}$$

(b) 99% CI for σ:

$$\left(\sqrt{0.3452},\ \sqrt{1.6729}\right) = \boxed{(0.5876,\ 1.2934)}$$

Scoring Rubric

Error	Deduction
Swaps upper/lower χ^2 values (inverted interval)	-5
Wrong χ^2 values from table	-3
Uses n instead of $n-1$ in numerator	-3
Part (b): forgets to take square root	-2
Arithmetic error	-2
Missing or no work shown	-12

Problem 5 [10 pt] — Sample Size Determination

Given: $\sigma^2 = 25$ ($\sigma = 5$), error $E = 2$, 95% confidence.

Solution:

Step 1: $n = \left(\dfrac{z_{\alpha/2}\,\sigma}{E}\right)^2 = \left(\dfrac{1.96 \times 5}{2}\right)^2 = (4.9)^2 = 24.01.$

Step 2: Round up: $\boxed{n = 25}$.

Scoring Rubric

Error	Deduction
Uses $\sigma^2 = 25$ instead of $\sigma = 5$	-4
Wrong formula structure	-3
Does not round up to next integer	-1
Arithmetic error	-2
Missing or no work shown	-8

Problem 6 [12 pt] — Large-Sample CI for the Mean

Given: $n = 81$, $\bar{x} = 42.8$ m, $s = 5.4$ m, 99% CI.

Solution:

Step 1: Since $n = 81$ is large, use z with s in place of σ.

Step 2: Standard error: $\text{SE} = 5.4/\sqrt{81} = 0.60$.

Step 3: 99% CI ($z_{0.005} = 2.576$):

$$42.8 \pm 2.576(0.60) = 42.8 \pm 1.546 \quad \Longrightarrow \quad \boxed{(41.25,\ 44.35)}$$

Scoring Rubric

Error	Deduction
Uses t instead of z (acceptable but must justify)	-1
Wrong z-value for 99%	-2
Wrong SE computation	-3
Arithmetic error	-2
Missing or no work shown	-10

Problem 7 [12 pt] — One-Sided Upper Confidence Bound

Given: $\sigma = 8$ ft/hr, $n = 20$, $\bar{x} = 65$ ft/hr, 90% UCB.

Solution:

Step 1: Standard error: $\text{SE} = 8/\sqrt{20} = 1.7889$.

Step 2: For a one-sided 90% bound, $z_{0.10} = 1.282$.

Step 3: Upper confidence bound:

$$\mu \leq \bar{x} + z_{0.10} \cdot \text{SE} = 65 + 1.282(1.7889) = 65 + 2.293 \quad \Longrightarrow \quad \boxed{\mu \leq 67.29}$$

Scoring Rubric

Error	Deduction
Constructs two-sided CI instead of one-sided bound	-4
Uses $z_{0.05} = 1.645$ instead of $z_{0.10} = 1.282$	-3
Computes lower bound instead of upper bound	-3
Arithmetic error	-2
Missing or no work shown	-10

Problem 8 [12 pt] — Confidence Interval for a Proportion

Given: $n = 120$, $x = 18$, 90% CI.

Solution:

Step 1: Point estimate: $\hat{p} = 18/120 = 0.15$.

Step 2: Standard error: $\text{SE} = \sqrt{0.15 \times 0.85/120} = 0.0326$.

Step 3: 90% CI ($z_{0.05} = 1.645$):

$$0.15 \pm 1.645(0.0326) = 0.15 \pm 0.0536 \quad \Longrightarrow \quad \boxed{(0.0964,\ 0.2036)}$$

Scoring Rubric

Error	Deduction
Wrong $\hat{p}$ computation	-3
Wrong SE formula for proportion	-4
Wrong z-value	-2
Arithmetic error	-2
Missing or no work shown	-10

Set A Total: $12 + 15 + 12 + 15 + 10 + 12 + 12 + 12 =$ **100 points.**

Set B

Problem 1 [12 pt] — Large-Sample CI for the Mean

Given: $n = 40$, $\bar{x} = 0.85$ mg/L, $s = 0.15$ mg/L, 95% CI.

Solution:

Step 1: Since $n = 40$ is large, use z with s in place of σ.

Step 2: Standard error: $\text{SE} = 0.15/\sqrt{40} = 0.0237$.

Step 3: 95% CI ($z_{0.025} = 1.96$):

$$0.85 \pm 1.96(0.0237) = 0.85 \pm 0.0465 \quad \Longrightarrow \quad \boxed{(0.8035,\ 0.8965)}$$

Scoring Rubric

Error	Deduction
Uses t instead of z (acceptable but must justify)	-1
Wrong z-value	-2
Wrong SE computation	-3
Arithmetic error	-2
Missing or no work shown	-10

Problem 2 [15 pt] — Chi-Square Interval for Variance

Given: $n = 18$, $s = 2.4$ N·m, $s^2 = 5.76$, $\nu = 17$, 95% CI.

Solution:

Step 1: From the χ^2 table with $\nu = 17$: $\chi^2_{0.025,17} = 30.191$ and $\chi^2_{0.975,17} = 7.564$.

(a) 95% CI for σ^2:

$$\left(\frac{17 \times 5.76}{30.191},\ \frac{17 \times 5.76}{7.564}\right) = \left(\frac{97.92}{30.191},\ \frac{97.92}{7.564}\right) = \boxed{(3.243,\ 12.945)}$$

(b) 95% CI for σ:

$$\left(\sqrt{3.243},\ \sqrt{12.945}\right) = \boxed{(1.801,\ 3.598)}$$

Scoring Rubric

Error	Deduction
Swaps upper/lower χ^2 values (inverted interval)	-5
Wrong χ^2 values from table	-3
Uses n instead of $n-1$ in numerator	-3
Part (b): forgets to take square root	-2
Arithmetic error	-2
Missing or no work shown	-12

Problem 3 [12 pt] — Confidence Interval for a Proportion

Given: $n = 180$, $x = 171$, 99% CI.

Solution:

Step 1: Point estimate: $\hat{p} = 171/180 = 0.95$.

Step 2: Standard error: $\text{SE} = \sqrt{0.95 \times 0.05/180} = 0.0162$.

Step 3: 99% CI ($z_{0.005} = 2.576$):

$$0.95 \pm 2.576(0.0162) = 0.95 \pm 0.0418 \quad \Longrightarrow \quad \boxed{(0.9082,\ 0.9918)}$$

Scoring Rubric

Error	Deduction
Wrong $\hat{p}$ computation	-3
Wrong SE formula for proportion	-4
Wrong z-value for 99%	-2
Arithmetic error	-2
Missing or no work shown	-10

Problem 4 [10 pt] — Sample Size Determination

Given: $\sigma = 3.5$, error $E = 1$, 90% confidence.

Solution:

Step 1: $n = \left(\dfrac{z_{\alpha/2}\,\sigma}{E}\right)^2 = \left(\dfrac{1.645 \times 3.5}{1}\right)^2 = (5.7575)^2 = 33.15.$

Step 2: Round up: $\boxed{n = 34}$.

Scoring Rubric

Error	Deduction
Wrong z-value for 90%	-3
Wrong formula structure	-3
Does not round up to next integer	-1
Arithmetic error	-2
Missing or no work shown	-8

Problem 5 [12 pt] — T-Interval for the Mean (Unknown σ)

Given: $n = 10$, $\bar{x} = 4.2$ m/s^2, $s = 0.6$ m/s^2, 99% CI.

Solution:

Step 1: Degrees of freedom: $\nu = 9$. For 99% CI, $t_{0.005,9} = 3.250$.

Step 2: Standard error: $\text{SE} = 0.6/\sqrt{10} = 0.1897$.

Step 3: Margin of error: $3.250 \times 0.1897 = 0.6166$.

$$4.2 \pm 0.617 \quad \Longrightarrow \quad \boxed{(3.583,\ 4.817)}$$

Scoring Rubric

Error	Deduction
Uses z instead of t (unknown σ)	-4
Wrong degrees of freedom	-2
Wrong t-value from table	-2
Arithmetic error	-2
Missing or no work shown	-10

Problem 6 [15 pt] — Z-Interval and Lower Confidence Bound

Given: $\sigma = 450$ bbl/day, $n = 25$, $\bar{x} = 12{,}500$ bbl/day.

Solution:

Standard error: $\text{SE} = 450/\sqrt{25} = 90$.

(a) 95% two-sided CI ($z_{0.025} = 1.96$):

$$12{,}500 \pm 1.96(90) = 12{,}500 \pm 176.4 \quad \Longrightarrow \quad \boxed{(12{,}323.60,\ 12{,}676.40)}$$

(b) 95% lower confidence bound ($z_{0.05} = 1.645$):

$$\mu \geq 12{,}500 - 1.645(90) = 12{,}500 - 148.05 \quad \Longrightarrow \quad \boxed{\mu \geq 12{,}351.95}$$

Scoring Rubric

Error	Deduction
Wrong SE computation	-3
Part (a): wrong z-value for 95%	-2
Part (b): uses two-sided z for one-sided bound	-3
Part (b): gives upper bound instead of lower	-3
Arithmetic error	-2
Missing or no work shown	-12

Problem 7 [12 pt] — Confidence Interval for a Proportion

Given: $n = 95$, $x = 7$, 90% CI.

Solution:

Step 1: Point estimate: $\hat{p} = 7/95 = 0.0737$.

Step 2: Standard error: $\text{SE} = \sqrt{0.0737 \times 0.9263/95} = 0.0268$.

Step 3: 90% CI ($z_{0.05} = 1.645$):

$$0.0737 \pm 1.645(0.0268) = 0.0737 \pm 0.0441 \quad \Longrightarrow \quad \boxed{(0.0296,\ 0.1178)}$$

Scoring Rubric

Error	Deduction
Wrong $\hat{p}$ computation	-3
Wrong SE formula for proportion	-4
Wrong z-value for 90%	-2
Arithmetic error	-2
Missing or no work shown	-10

Problem 8 [12 pt] — Sample Size Determination

Given: $\sigma^2 = 100$ ($\sigma = 10$), error $E = 3$, 99% confidence.

Solution:

Step 1: $n = \left(\dfrac{z_{\alpha/2}\,\sigma}{E}\right)^2 = \left(\dfrac{2.576 \times 10}{3}\right)^2 = (8.587)^2 = 73.73.$

Step 2: Round up: $\boxed{n = 74}$.

Scoring Rubric

Error	Deduction
Uses $\sigma^2 = 100$ instead of $\sigma = 10$	-4
Wrong formula structure	-4
Does not round up to next integer	-2
Arithmetic error	-2
Missing or no work shown	-10

Set B Total: $12 + 15 + 12 + 10 + 12 + 15 + 12 + 12 =$ **100 points.**