ISE 315: Engineering Statistics

Supplementary Material: Reading Z-Tables & Excel Formulas

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Based on Montgomery & Runger, Applied Statistics and Probability for Engineers, 6th Ed.

Overview

- Part 1: Reading the Standard Normal (Z) Table
  - Finding $P(Z \leq z)$ from the table
  - Finding z from a given probability
- Part 2: Excel Formulas for Normal Distribution
  - `NORM.S.DIST` and `NORM.S.INV` for Z
  - `NORM.DIST` and `NORM.INV` for $\bar{X}$
- Part 3: Examples
  - Z-score examples (from Lecture 2)
  - Non-standardized $\bar{X}$ example
  - Confidence intervals at 80%, 95%, 99.9999%
- Part 4: Practice Problems

Part 1

Reading the Standard Normal (Z) Table

Structure of the Z-Table

- The Z-table gives $\Phi(z) = P(Z \leq z)$ for $Z \sim N(0, 1)$
- Rows: first digit and first decimal of z (e.g., 1.9)
- Columns: second decimal of z (e.g., 0.06)

z	**0.00**	**0.01**	**0.02**	$\cdots$	**0.05**	**0.06**	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	
1.6	0.9452	0.9463	0.9474	$\cdots$	0.9505	0.9515	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	
1.9	0.9713	0.9719	0.9726	$\cdots$	0.9744	0.9750	$\cdots$
2.0	0.9772	0.9778	0.9783	$\cdots$	0.9798	0.9803	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	

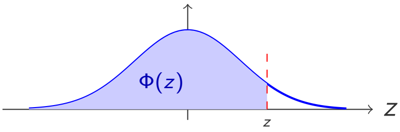
Example: $\Phi(1.96) = P(Z \leq 1.96) = 0.9750$

Task 1: Finding Probability from Z-Score

Given: A z-score, find $P(Z \leq z)$, $P(Z > z)$, or $P(a < Z < b)$

Key relationships:

- $P(Z \leq z) = \Phi(z)$ (read directly from table)
- $P(Z > z) = 1 - \Phi(z)$ (complement)
- $P(Z < -z) = 1 - \Phi(z)$ (symmetry)
- $P(a < Z < b) = \Phi(b) - \Phi(a)$ (subtraction)
- $P(|Z| < z) = 2\Phi(z) - 1$ (both tails)



Task 2: Finding Z-Score from Probability (Inverse)

Given: A probability p, find z such that $P(Z \leq z) = p$

Method: Search *inside* the table for p, then read row + column headers

Example: Find z such that $P(Z \leq z) = 0.975$

z	**0.00**	$\cdots$	**0.05**	**0.06**	**0.07**	$\cdots$
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	
1.8	0.9641	$\cdots$	0.9678	0.9686	0.9693	$\cdots$
1.9	0.9713	$\cdots$	0.9744	**0.9750**	0.9756	$\cdots$
2.0	0.9772	$\cdots$	0.9798	0.9803	0.9808	$\cdots$
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	

Common Critical Values

Conf. Level	α	Two-sided (CI)		One-sided	
		$\Phi(z_{\alpha/2})$	$z_{\alpha/2}$	$\Phi(z_\alpha)$	z_α
90%	0.10	0.95	1.645	0.90	1.282
95%	0.05	0.975	1.960	0.95	1.645
99%	0.01	0.995	2.576	0.99	2.326

How to read this table:

- **Two-sided:** Use $z_{\alpha/2}$ (e.g., 95% CI $\Rightarrow \alpha = 0.05 \Rightarrow z_{0.025} = 1.96$)
- **One-sided:** Use z_α (e.g., 95% lower bound $\Rightarrow \alpha = 0.05 \Rightarrow z_{0.05} = 1.645$)

Part 2

Excel Formulas for Normal Distribution

Excel Functions for Normal Distribution

For Standard Normal $Z \sim N(0, 1)$:

- `=NORM.S.DIST(z, TRUE)` $\Rightarrow$ Returns $\Phi(z) = P(Z \leq z)$
- `=NORM.S.INV(p)` $\Rightarrow$ Returns z such that $P(Z \leq z) = p$

For General Normal $X \sim N(\mu, \sigma)$:

- `=NORM.DIST(x,` μ`,` σ`, TRUE)` $\Rightarrow$ Returns $P(X \leq x)$
- `=NORM.INV(p,` μ`,` σ`)` $\Rightarrow$ Returns x such that $P(X \leq x) = p$

Important: The `TRUE` parameter gives CDF/cumulative probability.
`FALSE` gives the PDF—rarely needed for our class.
`NORM` uses standard deviation σ, NOT variance σ^2.

Excel Formula Quick Reference

Task	Standard Normal	General Normal
$P(Z \leq z)$ or $P(X \leq x)$	`NORM.S.DIST(z,TRUE)`	NORM.DIST(x,μ,σ,TRUE)
$P(Z > z)$ or $P(X > x)$	`1-NORM.S.DIST(z,TRUE)`	1-NORM.DIST(x,μ,σ,TRUE)
Find z given $P(Z \leq z) = p$	`NORM.S.INV(p)`	NORM.INV(p,μ,σ)

For Sampling Distribution of $\bar{X}$:

- Use `NORM.DIST` with $\mu_{\bar{X}} = \mu$ and $\sigma_{\bar{X}} = \sigma/\sqrt{n}$
- Example: `=NORM.DIST(0.6, 0.5, 0.091, TRUE)`

Part 3

Examples

From Lecture 2: Population $X_i \sim \text{Uniform}(0,1)$, sample size $n = 10$. Find $P(\bar{X} > 0.6)$.

Example 1: Standardized Z-Score

Step 1: Find population parameters

- $\mu = 0.5$, $\sigma^2 = 1/12 \Rightarrow \sigma = 0.289$

Step 2: Find sampling distribution of $\bar{X}$

- $\mu_{\bar{X}} = \mu = 0.5$
- $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{0.289}{\sqrt{10}} = 0.091$

Step 3: Standardize $Z = \frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}} = \frac{0.6 - 0.5}{0.091} = \boxed{1.10}$

Step 4: Find $P(\bar{X} > 0.6) = P(Z > 1.10) = 1 - \Phi(1.10)$ **(use table or Excel)**

Reading the Z-table for $z = 1.10$:

- Row: 1.1, Column: 0.00

z	**0.00**	**0.01**	**0.02**	$\cdots$
1.0	0.8413	0.8438	0.8461	$\cdots$
1.1	0.8643	0.8665	0.8686	$\cdots$
1.2	0.8849	0.8869	0.8888	$\cdots$

Excel alternative:

- `=NORM.S.DIST(1.10, TRUE)` returns `0.8643`

Answer: $P(\bar{X} > 0.6) = P(Z > 1.10) = 1 - \Phi(1.10) = 1 - 0.8643 = \boxed{0.1357}$

From Lecture 4: Two catalysts with $n_1 = 50$, $\sigma_1 = 4$ and $n_2 = 45$, $\sigma_2 = 5$. Find $P(|(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)| < 1.5)$.

Example 2: Difference of Means Z-Score

Step 1: Find standard error of $\bar{X}_1 - \bar{X}_2$

$$\text{SE} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = \sqrt{\frac{16}{50} + \frac{25}{45}} = \sqrt{0.876} = 0.936$$

Step 2: Standardize $Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\text{SE}} \quad \Rightarrow \quad |Z| < \frac{1.5}{0.936} = \boxed{1.60}$

Step 3: Find probability

$$P(|Z| < 1.60) = P(-1.60 < Z < 1.60) = \Phi(1.60) - \Phi(-1.60)$$

Example 2: Z-Table

Z-Table lookup for $z = 1.60$:

z	**0.00**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599

- Row: 1.6, Column: 0.00 $\Rightarrow \Phi(1.60) = 0.9452$
- By symmetry: $\Phi(-1.60) = 1 - \Phi(1.60) = 1 - 0.9452 = 0.0548$

Answer: $P(|Z| < 1.60) = \Phi(1.60) - \Phi(-1.60) = 0.9452 - 0.0548 = \boxed{0.8904}$

Drilling mud viscosity: $\mu = 48$ cp, $\sigma = 6$ cp, $n = 36$. Find $P(47 < \bar{X} < 50)$ using Excel (without standardizing).

Example 3: Non-Standardized $\bar{X}$ in Excel

Step 1: Identify sampling distribution parameters

- $\mu_{\bar{X}} = \mu = 48 \quad \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{6}{\sqrt{36}} = 1$

Step 2: Use NORM.DIST directly (no need to standardize!)

$$P(47 < \bar{X} < 50) = P(\bar{X} < 50) - P(\bar{X} < 47)$$

Excel formula:

- =NORM.DIST(x, μ, σ, TRUE) - NORM.DIST(x, μ, σ, TRUE)
- =NORM.DIST(50, 48, 1, TRUE) - NORM.DIST(47, 48, 1, TRUE)
- = 0.9772 - 0.1587 = $\boxed{0.8186}$

Pipeline wall thickness: $n = 25$, $\bar{x} = 0.2731$ in, $\sigma = 0.02$ in. Find CI at 80%, 95%, and 99.9999% confidence.

Example 4: Confidence Intervals at Different Levels

Step 1: Calculate standard error (same for all)

$$\text{SE} = \frac{\sigma}{\sqrt{n}} = \frac{0.02}{\sqrt{25}} = 0.004$$

Step 2: Find critical values

- 80% CI: $\alpha = 0.20 \Rightarrow z_{0.10} = 1.282$
- 95% CI: $\alpha = 0.05 \Rightarrow z_{0.025} = 1.960$
- 99.9999% CI: $\alpha = 0.000001 \Rightarrow z = ?$

Excel for 99.9999%: `=NORM.S.INV(0.9999995)` returns `4.8916`

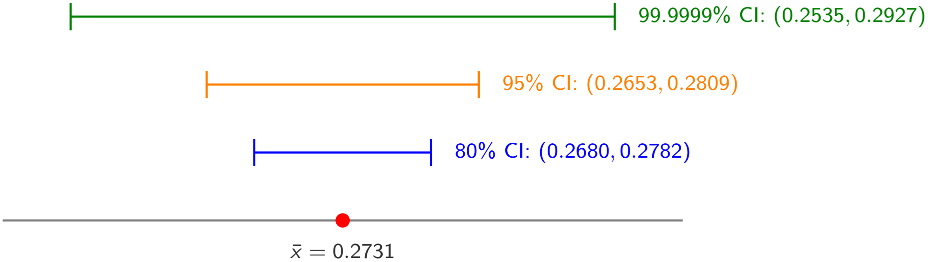
Example 4: Results Comparison

CI Formula: $\bar{x} \pm E = \bar{x} \pm z_{\alpha/2} \times \text{SE} = 0.2731 \pm z_{\alpha/2} \times 0.004$

Conf. Level	$z_{\alpha/2}$	E	**95% CI**
80%	1.282	$1.282 \times 0.004 = 0.00513$	$(0.2680, 0.2782)$
95%	1.960	$1.960 \times 0.004 = 0.00784$	$(0.2653, 0.2809)$
99.9999%	4.892	$4.892 \times 0.004 = 0.01957$	$(0.2535, 0.2927)$

- 80% CI is narrow but we're less confident
- 99.9999% CI is very wide but we're almost certain μ is inside

Visualizing Different Confidence Levels



Trade-off: Higher confidence = Wider interval

Practice Problems (Z-table and Excel)

1. Find $P(Z < 2.33)$ and $P(Z > -1.5) \Rightarrow \boxed{0.9901}, \boxed{0.9332}$
2. Find z such that $P(Z > z) = 0.01 \Rightarrow \boxed{2.326}$
3. If $X \sim N(100, 15^2)$ and $n = 25$, find $P(\bar{X} > 105) \Rightarrow \boxed{0.0478}$
4. Construct 90%, 95%, and 99.99% CIs for μ given:
   $n = 40$, $\bar{x} = 72.5$, $\sigma = 8$
   $\Rightarrow \boxed{(70.42, 74.58)}, \boxed{(70.02, 74.98)}, \boxed{(69.58, 75.42)}$